Parameterized Intuitionistic Fuzzy Optimization Method and its Application to Structural Design

Samir Dey
Department of Mathematics,
Asansol Engineering College
Vivekananda Sarani, Asansol-713305, West Bengal, India.
samir_besus@rediffmail.com

Tapan Kumar Roy
Department of Mathematics,
Indian Institute of Engineering Science and Technology, Shibpur.
P.O.-Botanic Garden, Howrah-711103, West Bengal, India.
roy_t_k@yahoo.co.in

Abstract— **This paper develops a solution procedure of multi-objective intuitionistic fuzzy optimization to solve a non-linear model with inexact co-efficient and resources. Interval approximation method is used here to convert the imprecise co-efficient which is a triangular fuzzy number to an interval number. We transform this interval number to a parametric interval valued functional form and then solve this parametric problem by intuitionistic fuzzy optimization technique. Usually interval valued optimization consist of two level mathematical programs but a parametric interval valued optimization in intuitionistic fuzzy environment is direct approach to find the objective function, this is the main advantage. In this paper we have considered a multi objective structural optimization model with weight and deflection as objectives and stress as constraint function. Numerical example is given here to illustrate this structural model through this approximation method.**

Keywords- Intuitionistic Fuzzy Set; Intuitionistic Fuzzy Number; Interval Valued Function; Structural Optimization.

I. INTRODUCTION

Optimization is a technique that deal with the problem of minimizing or maximizing a certain function in a finite dimensional Euclidean space over a subset of that space, which is determined by functional inequalities .It has been seen that numerous engineering design problem need to deal with noisy data, manufacturing error or uncertainty of the environment during the design process. Fuzzy as well as intuitionistic fuzzy optimization in case of structural engineering not only helps the engineers in their design and analysis of systems but also leads to significant advances and new discoveries in fuzzy optimization theory and technique. This fuzzy set theory was first introduced by Zadeh[5]. As an extension Intuitionistic fuzzy set theory was first introduced by Atanassove [4].When an imprecise information cannot be expressed by means of conventional fuzzy set Intuitionistic Fuzzy set play an important role. In intuitionistic fuzzy (IF) set we usually consider degree of acceptance, degree of non-membership and hesitancy function whereas we consider only membership function in fuzzy set. A few research works has been done on intuitionisticfuzzy optimization in the field of structural optimization. Dey et al.[3] used intuitionistic fuzzy technique to optimize single objective two bar truss structural model. Dubey et al.[1] introduced an algorithm to solve intuitionistic linear programming with imprecise co-efficient. Singh et al.[7] introduced an algorithm to solve multi-objective intuitionistic nonlinear programming problem. This is the first time a parametric-intuitionistic multi-objective nonlinear programming is introduced in this paper with an application in structural design.

The present study investigates computational algorithm for solving multi-objective non linear programming problem by parametric Intuitionistic fuzzy optimization approach. The remainder of this paper is organized in the following manner. In section II, we discuss about multi-objective structural model. In section III, we discuss about fuzzy set, intuitionistic fuzzy set, Intuitionistic fuzzy number, α -cut and arithmetic operation on triangular intuitionistic fuzzy number. In section IV, we discuss the Solution procedure of multi-objective nonlinear programming problem by parametric intuitionistic non-linear programming technique. In section V, we discuss about Solution of Multi-objective structural optimization Problem parametric intuitionistic fuzzy optimization technique. In section VI, we discuss about numerical solution of structural model of three bar truss. Finally, we draw conclusions from the results in section VII.

II. MULTI-OBJECTIVE STRUCTURAL MODEL

In the design problem of the structure i.e lightest weight of the structure and minimum deflection of the loaded joint that satisfies all stress constraints in members of the structure. In truss structure system ,the basic parameters (including allowable stress ,etc) are known and the optimization's target is that identify the optimal bar truss cross-section area so that the structure is of the smallest total weight with minimum nodes displacement in a given load conditions .

The multi-objective structural model can be expressed as

$$Minimize\ \ WT(A) \tag{1}$$

$$Minimize\ \ \delta(A)$$

$$subject\ to\ \sigma(A) \le [\sigma]$$

$$A^{\min} \le A \le A^{\max}$$

Where $A = [A_1, A_2, ...A_n]^T$ are the design variables for the cross section, n is the group number of design variables for the cross section bar , $WT(A) = \sum_{i=1}^{n} \rho_i A_i L_i$ is the total weight of the structure , $\delta(A)$ is the deflection of the loaded joint ,where L_i, A_i and ρ_i are the bar length ,cross section area and density of the i^{th} group bars respectively. $\sigma(A)$ is the stress constraint and $[\sigma]$ is allowable stress of the group bars under various conditions, $A^{\min}$ and $A^{\max}$ are the lower and upper bounds of cross section area A respectively.

III. MATHEMATICAL PRELIMINARIES

A. *Fuzzy Set*

Let X denotes a universal set. Then the fuzzy subset A in X is a subset of order pairs $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\}$ where $\mu_{\tilde{A}} : X \to [0,1]$ is called the membership function which assigns a real number $\mu_{\tilde{A}}(x)$ in the interval $[0,1]$ to each element $x \in X$. A is non-fuzzy and $\mu_{\tilde{A}}(x)$ is identical to the characteristic function of crisp set.It is clear that the range of membership function is a subset of non-negative real numbers.

B. *$\alpha-$Level Set or $\alpha-$ cut of a Fuzzy Set*

The α – level set of a fuzzy set A of X is a crisp set A_α which contains all the elements of X that have membership values greater than or equal to α i.e $A = \{x : \mu_A(x) \ge \alpha, x \in X, \alpha \in [0,1]\}$.

C. *Intuitionistic Fuzzy Set*

Let $X = \{x_1, x_2, ..., x_n\}$ be a finite universal set. An intuitionistic fuzzy set (IFS) set $\tilde{A}^i$ in the sense of Atanassove [4] is given by equation $\tilde{A}^i = \{< X, \mu_{\tilde{A}^i}(x), \upsilon_{\tilde{A}^i}(x) > | x_i \in X\}$ where the function $\mu_{\tilde{A}^i}(x^i) : X \to [0,1]$; $x_i \in X \to \mu_{\tilde{A}^i}(x_i) \in [0,1]$ and $\upsilon_{\tilde{A}^i}(x^i) : X \to [0,1]$; $x_i \in X \to \upsilon_{\tilde{A}^i}(x_i) \in [0,1]$ define the degree of membership and degree of non-membership of an element $x_i \in X$ to the set $\tilde{A}^i \subseteq X$,such that they satisfy the condition $0 \le \mu_{\tilde{A}^i}(x_i) + \upsilon_{\tilde{A}^i}(x_i) \le 1$, $\forall\ x_i \in X$. For each IFS $\tilde{A}^i$ in X the amount $\Pi_{\tilde{A}^i}(x_i) = 1 - (\mu_{\tilde{A}^i}(x^i) + \upsilon_{\tilde{A}^i}(x^i))$ is called the degree of uncertainty (or hesitation) associated with the membership of elements $x_i \in X$ in $\tilde{A}^i$ we call it intuitionistic fuzzy index of $\tilde{A}^i$ with respect of an element $x_i \in X$.

D. *Generalized Intuitionistic Fuzzy Number*

A generalised intuitionistic fuzzy number $\tilde{A}^i$ can be defined as with the following properties

i)It is an intuitionistic fuzzy subset of real line.
ii)It is normal i.e there is any $x_0 \in R$ such that $\mu_{\tilde{A}^i}(x_0) = w(\in R)$ and $\upsilon_{\tilde{A}^i}(x_0) = \tau(\in R)$ for $w + \tau \le 1$; .
iii)It is a convex set for membership function $\mu_{\tilde{A}^i}(x)$ i.e
$\mu_{\tilde{A}^i}(\lambda x_1 + (1-\lambda)x_2) \ge \min(\mu_{\tilde{A}^i}(x_1), \mu_{\tilde{A}^i}(x_2))$ for all $x_1, x_2 \in R, \lambda \in [0, w]$.
iv)It is a concave set for membership function $\mu_{\tilde{A}^i}(x)$ i.e
$\mu_{\tilde{A}^i}(\lambda x_1 + (1-\lambda)x_2) \ge \max(\mu_{\tilde{A}^i}(x_1), \mu_{\tilde{A}^i}(x_2))$ for all $x_1, x_2 \in R, \lambda \in [\tau, 1]$.
v) $\mu_{\tilde{A}^i}$ is continuous mapping from R to the closed interval $[0, w]$ and $\upsilon_{\tilde{A}^i}$ is continuous mapping from R to the closed interval $[\tau, 1]$ and for $x_0 \in R$ the relation $\mu_{\tilde{A}^i} + \upsilon_{\tilde{A}^i} \le 1$ holds.

E. Generalized Triangular Intuitionistic Fuzzy Number

A generalized triangular intuitionistic fuzzy number $\tilde{A}^i = \left((a_1^\mu, a_2, a_3^\mu; w_a)(a_1^\upsilon, a_2, a_3^\upsilon; \tau_a)\right)$ is a IFN in R and can be defined with the following membership function and non-membership function as follows

$$\mu_{\tilde{A}^i} = \begin{cases} w_a \dfrac{x - a_1^\mu}{a_2 - a_1^\mu} & for\ a_1^\mu \le x \le a_2 \\ w_a & for\ x = a_2 \\ w_a \dfrac{a_3^\mu - x}{a_3^\mu - a_2} & for\ a_2 \le x \le a_3^\mu \\ 0 & otherwise \end{cases} \qquad \upsilon_{\tilde{A}^i} = \begin{cases} \tau_a \dfrac{x - a_1^\upsilon}{a_2 - a_1^\upsilon} & for\ a_1^\upsilon \le x \le a_2 \\ \sigma_a & for\ x = a_2 \\ \tau_a \dfrac{x - a_2}{a_3^\upsilon - a_2} & for\ a_2 \le x \le a_3^\upsilon \\ 1 & otherwise \end{cases}$$

where $a_1^\upsilon \le a_1^\mu \le a_2 \le a_3^\mu \le a_3^\upsilon$.

F. $\alpha-$ Level set or $\alpha-$ cut of a Intuitionistic Fuzzy Number

Let $\tilde{A}^i = \left((a_1^\mu, a_2, a_3^\mu; w_a)(a_1^\upsilon, a_2, a_3^\upsilon; \tau_a)\right)$ be a triangular intuitionistic fuzzy number then $\alpha-$ cut of this fuzzy number is defined by the closed interval $\left[\mu_{A_L^i}(\alpha), \mu_{A_U^i}(\alpha)\right], \alpha \in (0,1]$ and $\left[\upsilon_{A_L^i}(\alpha), \upsilon_{A_U^i}(\alpha)\right], \alpha \in [0,1)$ where $\mu_{A_L^i}(\alpha) = \inf\left\{x \in R : \mu_{A_L^i}(x) \ge \alpha\right\}$

$\mu_{A_U^i}(\alpha) = \sup\left\{x \in R : \mu_{A_U^i}(x) \ge \alpha\right\}$, $\upsilon_{A_L^i}(\alpha) = \inf\left\{x \in R : \upsilon_{A_L^i}(x) \le \alpha\right\}$ $\upsilon_{A_U^i}(\alpha) = \sup\left\{x \in R : \upsilon_{A_U^i}(x) \le \alpha\right\}$,
Maintaining the Integrity of the Specifications

G. Arithmetic Operations on Triangular Intuitionistic Fuzzy Numbers

Let $\tilde{A}^i = \left((a_1^\mu, a_2, a_3^\mu; w_a)(a_1^\upsilon, a_2, a_3^\upsilon; \tau_a)\right)$ and $\tilde{B}^i = \left((b_1^\mu, b_2, b_3^\mu; w_b)(b_1^\upsilon, b_2, b_3^\upsilon; \tau_b)\right)$ be two triangular intuitionistic fuzzy number then the arithmetic operations on these numbers can be defined as follows

(i) $\tilde{A}^i + \tilde{B}^i = \left((a_1^\mu + b_1^\mu, a_2 + b_2, a_3^\mu + b_3^\mu; \min(w_a, w_b))(a_1^\upsilon + b_1^\upsilon, a_2 + b_2, a_3^\upsilon + b_3^\upsilon; \max(\tau_a, \tau_b))\right)$

(ii) $\tilde{A}^i - \tilde{B}^i = \left((a_1^\mu - b_1^\mu, a_2 - b_2, a_3^\mu - b_3^\mu; \min(w_a, w_b))(a_1^\upsilon - b_1^\upsilon, a_2 - b_2, a_3^\upsilon - b_3^\upsilon; \max(\tau_a, \tau_b))\right)$

(iii) $k\tilde{A}^i = \begin{cases} \left((ka_1^\mu, ka_2, ka_3^\mu; w_a)(ka_1^\upsilon, ka_2, ka_3^\upsilon; \tau_a)\right) & for\ k > 0 \\ \left((ka_3^\mu, ka_2, ka_1^\mu; w_a)(ka_3^\upsilon, ka_2, ka_1^\upsilon; \tau_a)\right) & for\ k < 0 \end{cases}$

(iv) $\tilde{A}^i.\tilde{B}^i = \begin{cases} \left((a_1^\mu b_1^\mu, a_2 b_2, a_3^\mu b_3^\mu; \min(w_a, w_b))(a_1^\upsilon b_1^\upsilon, a_2 b_2, a_3^\upsilon b_3^\upsilon; \max(\tau_a, \tau_b))\right) & for\ \tilde{A}^i > 0, \tilde{B}^i > 0 \\ \left((a_1^\mu b_3^\mu, a_2 b_2, a_3^\mu b_1^\mu; \min(w_a, w_b))(a_1^\upsilon b_3^\upsilon, a_2 b_2, a_3^\upsilon b_1^\upsilon; \max(\tau_a, \tau_b))\right) & for\ \tilde{A}^i > 0, \tilde{B}^i < 0 \\ \left((a_3^\mu b_3^\mu, a_2 b_2, a_1^\mu b_1^\mu; \min(w_a, w_b))(a_3^\upsilon b_3^\upsilon, a_2 b_2, a_1^\upsilon b_1^\upsilon; \max(\tau_a, \tau_b))\right) & for\ \tilde{A}^i < 0, \tilde{B}^i < 0 \end{cases}$

$$\text{(v) } \tilde{A}^i/\tilde{B}^i = \begin{cases} \left(\left(a_1^\mu/b_3^\mu, a_2/b_2, a_3^\mu/b_1^\mu; \min\left(w_a, w_b\right)\right)\left(a_1^\upsilon/b_3^\upsilon, a_2/b_2, a_3^\upsilon/b_1^\upsilon; \max\left(\sigma_a, \sigma_b\right)\right)\right) & for\, \tilde{A}^i > 0, \tilde{B}^i > 0 \\ \left(\left(a_3^\mu/b_3^\mu, a_2/b_2, a_1^\mu/b_1^\mu; \min\left(w_a, w_b\right)\right)\left(a_3^\upsilon/b_3^\upsilon, a_2/b_2, a_1^\upsilon/b_1^\upsilon; \max\left(\sigma_a, \sigma_b\right)\right)\right) & for\, \tilde{A}^i < 0, \tilde{B}^i > 0 \\ \left(\left(a_3^\mu/b_1^\mu, a_2/b_2, a_1^\mu/b_3^\mu; \min\left(w_a, w_b\right)\right)\left(a_3^\upsilon/b_1^\upsilon, a_2/b_2, a_1^\upsilon/b_3^\upsilon; \max\left(\sigma_a, \sigma_b\right)\right)\right) & for\, \tilde{A}^i < 0, \tilde{B}^i < 0 \end{cases}$$

IV. MATHEMATICAL ANALYSIS

A. *Nearest Interval Approximation*

Here we want to approximate an intuitionistic fuzzy number $\tilde{A}^i = \left(\left(a_1^\mu, a_2, a_3^\mu; w_a\right)\left(a_1^\upsilon, a_2, a_3^\upsilon; \tau_a\right)\right)$ by a crisp model.

Let $\tilde{A}^i$ and $\tilde{B}^i$ be two intuitionistic fuzzy numbers. Then the distance between them can be measured according to Euclidean matric as

$$\tilde{d}_E^2 = \frac{1}{2}\int_0^1 \left(\mu_{A_L}(\alpha) - \mu_{B_L}(\alpha)\right)^2 d\alpha + \frac{1}{2}\int_0^1 \left(\mu_{A_U}(\alpha) - \mu_{B_U}(\alpha)\right)^2 d\alpha + \frac{1}{2}\int_0^1 \left(\upsilon_{A_L}(\alpha) - \upsilon_{B_L}(\alpha)\right)^2 d\alpha$$
$$+ \frac{1}{2}\int_0^1 \left(\upsilon_{A_U}(\alpha) - \upsilon_{B_U}(\alpha)\right)^2 d\alpha$$

Now we find a closed interval $\tilde{C}_{d_E}\left(\tilde{A}^i\right) = \left[C_L, C_U\right]$ which is nearest to $\tilde{A}^i$ with respect to the matric $\tilde{d}_E$ .Again it is obvious that each real interval can also be considered as an intuitionistic fuzzy number with constant $\alpha -$ cut $\left[C_L, C_U\right]$ for all $\alpha \in [0,1]$ .Now we have to minimize $\tilde{d}_E\left(\tilde{A}^i, \tilde{C}_{d_E}\left(\tilde{A}^i\right)\right)$ with respect to C_L and C_U ,that is to minimize

$$F_1\left(C_L, C_U\right) = \frac{1}{2}\int_0^1 \left(\mu_{A_L}(\alpha) - C_L\right)^2 d\alpha + \frac{1}{2}\int_0^1 \left(\mu_{A_U}(\alpha) - C_U\right)^2 d\alpha + \frac{1}{2}\int_0^1 \left(\upsilon_{A_L}(\alpha) - C_L\right)^2 d\alpha$$
$$+ \frac{1}{2}\int_0^1 \left(\upsilon_{A_U}(\alpha) - C_U\right)^2 d\alpha$$

With respect to C_L and C_U . We define partial derivatives $\frac{\partial F_1\left(C_L, C_U\right)}{\partial C_L} = -2\int_0^1 \left(\mu_{A_L}(\alpha) + \upsilon_{A_L}(\alpha)\right) d\alpha + 4C_L$

$$\frac{\partial F_1\left(C_L, C_U\right)}{\partial C_U} = -2\int_0^1 \left(\mu_{A_U}(\alpha) + \upsilon_{A_U}(\alpha)\right) d\alpha + 4C_U$$

And then we solve the system

$$\frac{\partial F_1\left(C_L, C_U\right)}{\partial C_L} = 0, \frac{\partial F_1\left(C_L, C_U\right)}{\partial C_U} = 0$$

The solution is

$$C_L = \int_0^1 \frac{\mu_{A_L}(\alpha) + \upsilon_{A_L}(\alpha)}{2} d\alpha;\ C_U = \int_0^1 \frac{\mu_{A_U}(\alpha) + \upsilon_{A_U}(\alpha)}{2} d\alpha$$

Since $\det\begin{pmatrix} \frac{\partial^2 F_1\left(C_L, C_U\right)}{\partial C_L^2} & \frac{\partial^2 F_1\left(C_L, C_U\right)}{\partial C_L \partial C_U} \\ \frac{\partial^2 F_1\left(C_L, C_U\right)}{\partial C_U \partial C_L} & \frac{\partial^2 F_1\left(C_L, C_U\right)}{\partial C_U^2} \end{pmatrix} = \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix} = 4 > 0$

then C_L C_U mentioned above minimize $F_1\left(C_L, C_U\right)$. The nearest interval of the intuitionistic fuzzy number $\tilde{A}^i$ with respect to the metric $\tilde{d}_E$ is

$$\tilde{C}_{d_E}\left(\tilde{A}^i\right)=\left[\int_0^1 \frac{\mu_{A_L}(\alpha)+\upsilon_{A_L}(\alpha)}{2}d\alpha, \int_0^1 \frac{\mu_{A_U}(\alpha)+\upsilon_{A_U}(\alpha)}{2}d\alpha\right]$$
$$=\left[\frac{a_1^{\mu}+a_2}{2}+\frac{a_2-a_1^{\mu}}{4w}+\frac{a_2-a_1^{\upsilon}}{4\tau}, \frac{a_2+a_3^{\mu}}{2}+\frac{a_3^{\mu}-a_2}{4w}+\frac{a_3^{\upsilon}-a_2}{4\tau}\right]$$

B. Parametric Interval Valued Function

If $[m,n]$ be an interval with $m,n>0$ we can express an interval number by a function. The parametric interval-valued function for the interval $[m,n]$ can be taken as $g(s)=m^{1-s}n^s$ for $s\in[0,1]$ which is strictly monotone continuous function and its inverse exists .Let ψ be the inverse of $g(s)$ then $s=\frac{\log\psi-\log m}{\log n-\log m}$.

C. Formulation of Intuitionistic Programming with Imprecise Coefficient in Parametric Form

A multi-objective intuitionistic fuzzy non-linear programming problem with imprecise co-efficient can be formulated as

$$\textit{Minimize } \tilde{f}_{k_0}(x)=\sum_{t=1}^{T_{k_0}}\xi_{k_0t}\tilde{c}_{k_0t}\prod_{j=1}^{n}x_j^{a_{k_0tj}} \text{ for } k_0=1,2,....,p \tag{2}$$

Such that $\tilde{f}_i(x)=\sum_{t=1}^{T_i}\xi_{it}\tilde{c}_{it}\prod_{j=1}^{n}x_j^{a_{itj}}\le\xi_i\tilde{b}_i$ for $i=1,2,....,m$

$x_j>0\ \ j=1,2,....,n$

Here ξ_{k_0t}, ξ_{it}, ξ_i are the signum function used to indicate sign of term in the equation. $\tilde{c}_{k_0t}>0,\ \tilde{c}_{it}>0$. a_{k_0tj}, a_{itj} are real numbers for all i,t,k_0,j.

Here $\tilde{c}_{k_0}=\left(\left(c_{k_0}^{1\mu},c_{k_0}^{2},c_{k_0}^{3\mu};w_{k_0}\right)\left(c_{k_0}^{1\upsilon},c_{k_0}^{2},c_{k_0}^{3\upsilon};\tau_{k_0}\right)\right)$;

$\tilde{c}_{it}=\left(\left(c_{it}^{1\mu},c_{it}^{2},c_{it}^{3\mu};w_{it}\right)\left(c_{it}^{1\upsilon},c_{it}^{2},c_{it}^{3\upsilon};\tau_{it}\right)\right)$; $\tilde{b}_i=\left(\left(b_i^{1\mu},b_i^{2},b_i^{3\mu};w_i\right)\left(b_i^{1\upsilon},b_i^{2},b_i^{3\upsilon};\tau_i\right)\right)$.

Using nearest interval approximation method ,we transform all the triangular intuitionistic fuzzy number into interval number i.e $\left[c_{k_0t}^L,c_{k_0t}^U\right]$, $\left[c_{it}^L,c_{it}^U\right]$, and $\left[b_i^L,b_i^U\right]$

Now the intuitionistic multi-objective programming with imprecise parameter is of the following form

$$\textit{Minimize } \hat{f}_{k_0}(x)=\sum_{t=1}^{T_{k_0}}\xi_{k_0t}\hat{c}_{k_0t}\prod_{j=1}^{n}x_j^{a_{k_0tj}} \text{ for } k_0=1,2,....,p$$

Such that $\hat{f}_i(x)=\sum_{t=1}^{T_i}\xi_{it}\hat{c}_{it}\prod_{j=1}^{n}x_j^{a_{itj}}\le\sigma_i\hat{b}_i$ for $i=1,2,....,m$

$x_j>0\ \ j=1,2,....,n$

Here ξ_{k_0t}, ξ_{it}, ξ_i are the signum function used to indicate sign of term in the equation. $\hat{c}_{k_0t}>0,\ \hat{c}_{it}>0;\ \hat{b}_i>0$ denote the interval component i.e $\hat{c}_{k_0t}=\left[c_{k_0t}^L,c_{k_0t}^U\right]$, $\hat{c}_{it}=\left[c_{it}^L,c_{it}^U\right]$, and $\hat{b}_i=\left[b_i^L,b_i^U\right]$ and a_{k_0tj}, a_{itj} are real numbers for all i,t,k_0,j.

Using parametric interval valued function the above problem transform into

$$\textit{Minimize } f_{k_0}(x;s)=\sum_{t=1}^{T_{k_0}}\xi_{k_0t}\left(c_{k_0t}^L\right)^{1-s}\left(c_{k_0t}^U\right)^{s}\prod_{j=1}^{n}x_j^{a_{k_0tj}} \text{ for } k_0=1,2,....,p \tag{3}$$

Such that $f_i(x;s)=\sum_{t=1}^{T_i}\xi_{it}\left(c_{it}^L\right)^{1-s}\left(c_{it}^U\right)^{s}\prod_{j=1}^{n}x_j^{a_{itj}}\le\xi_i\left(b_i^L\right)^{1-s}\left(b_i^U\right)^{s}$ for $i=1,2,....,m$

$x_j > 0 \;\; j = 1,2,....,n \;\; s \in [0,1]$

Here $\xi_{k_0 t}$, ξ_{it}, ξ_i are the signum function used to indicate sign of term in the equation.

This is a parametric multi-objective non-linear programming problem and can be solved by intuitionistic fuzzy optimization technique.

D. Intuitionistic Fuzzy Non-linear Programming (IFNLP) Optimization to solve Parametric Multi-Objective Non-linear Programming Problem (PMONLP)

A multi-objective non-linear parametric intuitionistic programming (MONLP) Problem can be formulated as

$$Minimize \;\left\{ f_1(x;s), f_2(x;s),...., f_p(x;s) \right\}^T \tag{4}$$

Subject to $g_j(x;s) \le b_j; \quad j = 1,2,.....,m$

$x > 0 \;\; s \in [0,1]$

Following Zimmermann [6],we have presented a solution algorithm to solve the MONLP Problem by fuzzy optimization technique.

Step-1: Solve the MONLP (4) as a single objective non-linear programming problem p th by taking one of the objective at a time and ignoring the others .These solutions are known as ideal solutions. Let x^i be the respective optimal solution for the i^{th} different objectives with same constraints and evaluate each objective values for all these i^{th} optimal solutions.

Step-2: From the result of step -1 determine the corresponding values for every objective for each derived solutions. With the values of all objectives at each ideal solutions ,pay-off matrix can be formulated as follows

$$\begin{array}{c} \\ x^1 \\ x^2 \\ \vdots \\ x^p \end{array} \begin{array}{c} \begin{array}{cccc} f_1(x;s) & f_2(x;s) & .... & f_p(x;s) \end{array} \\ \begin{bmatrix} f_1^*(x^1;s) & f_2^*(x^1;s) & .... & f_p^*(x^1;s) \\ f_1^*(x^2;s) & f_2^*(x^2;s) & .... & f_p^*(x^2;s) \\ .... & .... & .... & .... \\ f_1^*(x^p;s) & f_2^*(x^p;s) & .... & f_p^*(x^p;s) \end{bmatrix} \end{array}$$

Here $x^1, x^2,......, x^p$ are the ideal solution of the objectives $f_1(x;s), f_2(x;s),...., f_p(x;s)$ respectively.

Step-3: From the result of step 2 now we find lower bound (minimum) L_i^{ACC} and upper bound (maximum) U_i^{ACC} by using following rule $U_i^{ACC} = \max\left\{ f_i\left(x^p;s\right)\right\}, L_i^{ACC} = \min\left\{ f_i\left(x^p;s\right)\right\}$ where $1 \le i \le p$ .But in IFO The degree of non-membership (rejection) and the degree of membership (acceptance) are considered so that the sum of both value is less than one. To define the non -membership of NLP problem let $U_i^{\mathrm{Re}j}$ and $L_i^{\mathrm{Re}j}$ be the upper bound and lower bound of objective function $f_i(x)$ where $L_i^{ACC} \le L_i^{\mathrm{Re}j} \le U_i^{\mathrm{Re}j} \le U_i^{ACC}$ .For objective function of minimization problem ,the upper bound for non-membership function (rejection) is always equals to that the upper bound of membership function (acceptance).One can take lower bound for non-membership function as follows $L_i^{\mathrm{Re}j} = L_i^{Acc} + \varepsilon_i$ where $0 < \varepsilon_i < \left(U_i^{Acc} - L_i^{Acc}\right)$ based on the decision maker choice.

The initial intuitionistic fuzzy model with aspiration level of objectives becomes *Find* $\{x_i, i = 1,2,...., p\}$ so as to satisfy $f_i(x) \le^i L_i^{Acc}$ with tolerance $P_i^{Acc} = \left(U_i^{Acc} - L_i^{Acc}\right)$ for the degree of acceptance for $i = 1,2,....., p$. $f_i(x;s) \ge^i U_i^{\mathrm{Re}j}$ with tolerance $P_i^{Acc} = \left(U_i^{Acc} - L_i^{Acc}\right)$ for degree of rejection for $i = 1,2,....., p$ .Define the membership (acceptance) and non-membership (rejection) functions of above uncertain objectives as follows. For the $i^{th}, i = 1,2,...., p$ objectives functions the linear membership function $\mu_i\left(f_i(x;s)\right)$ and linear non-membership $\upsilon_i\left(f_i(x;s)\right)$ is defined as follows

$$\mu_i\left(f_i(x;s)\right)=\begin{cases} 1 & if \quad f_i(x;s)\le L_i^{Acc} \\ \dfrac{e^{-T\left(\frac{f_i(x;s)-L_i^{Acc}}{U_i^{Acc}-L_i^{Acc}}\right)}-e^{-T}}{1-e^{-T}} & if \quad L_i^{Acc}\le f_i(x;s)\le U_i^{Acc} \\ 0 & if \quad f_i(x;s)\ge U_i^{Acc} \end{cases}$$

$$\upsilon_i\left(f_i(x;s)\right)=\begin{cases} 0 & if \quad f_i(x;s)\le L_i^{\mathrm{Re}\,j} \\ \left(\dfrac{f_i(x;s)-L_i^{\mathrm{Re}\,j}}{U_i^{\mathrm{Re}\,j}-L_i^{\mathrm{Re}\,j}}\right)^2 & if \quad L_i^{\mathrm{Re}\,j}\le f_i(x;s)\le U_i^{\mathrm{Re}\,j} \\ 1 & if \quad f_i(x;s)\ge U_i^{\mathrm{Re}\,j} \end{cases}$$

Step-4: Now an Intuitionistic fuzzy optimization for above problem with membership and non-membership function can be written as

$$\begin{aligned} &\underset{\forall i}{Maximize}\left(\mu_i\left(f_i(x;s)\right)\right) \\ &\underset{\forall i}{Minimize}\left(\upsilon_i\left(f_i(x;s)\right)\right) \\ &subject\ to\ \mu_i\left(f_i(x;s)\right)+\upsilon_i\left(f_i(x;s)\right)<1 \\ &\left(\mu_i\left(f_i(x;s)\right)\right)>\left(\upsilon_i\left(f_i(x;s)\right)\right); \\ &\left(\upsilon_i\left(f_i(x;s)\right)\right)\ge 0; \\ &g_j(x;s)\le b_j; \\ &x>0\ \ s\in[0,1] \\ &i=1,2,\ldots..,p;\ j=1,2,\ldots..,m \end{aligned} \tag{5}$$

Find an equivalent crisp model by using membership and non-membership functions of objectives by IF as follows

$$\begin{aligned} &Max\left(Min\left(\mu_1,\mu_2,\ldots.,\mu_p\right)\right)-Min\left(Max\left(\upsilon_1,\upsilon_2,\ldots.,\upsilon_p\right)\right) \\ &subject\ to\ \mu_i\left(f_i(x;s)\right)+\upsilon_i\left(f_i(x;s)\right)<1 \\ &\left(\mu_i\left(f_i(x;s)\right)\right)>\left(\upsilon_i\left(f_i(x;s)\right)\right); \\ &\left(\upsilon_i\left(f_i(x;s)\right)\right)\ge 0; \\ &g_j(x;s)\le b_j; \\ &x>0 \\ &i=1,2,\ldots..,p;\ j=1,2,\ldots.,m \end{aligned} \tag{6}$$

If we consider $\alpha=Minimize\left(\mu_1,\mu_2,\ldots,\mu_p\right);\ \beta=Maximize\left(\upsilon_1,\upsilon_2,\ldots,\upsilon_p\right)$ accordingly the Angelov [2], the above can be written as

$$\begin{aligned} &Maximize\ \left(\alpha-\beta\right) \\ &subject\ to\ \mu_i\left(f_i(x;s)\right)\ge\alpha; \\ &g_j(x;s)\le 0; \\ &x>0,\alpha+\beta\le 1 \\ &s\in[0,1] \\ &\alpha\in[0,1],\beta\in[0,1];\ i=1,2,\ldots.,p \\ &j=1,2,\ldots..,m \end{aligned} \tag{7}$$

which on substitution of $\mu_i\left(f_i(x;s)\right)$ *and* $\upsilon_i\left(f_i(x;s)\right)$ *for* $i=1,2,\ldots,p$ becomes

$$Maximize\ (\alpha-\beta) \tag{8}$$

subject to

$$f_i(x;s)+\frac{U_i^{Acc}-L_i^{Acc}}{T}\ln\left\{\left(1-e^{-T}\right)\alpha+e^{-T}\right\}\le L_i^{Acc};$$
$$f_i(x;s)-\sqrt{\beta}\left(U_i^{\mathrm{Re}j}-L_i^{\mathrm{Re}j}\right)\le L_i^{\mathrm{Re}j};$$
$$g_j(x;s)\le 0;$$
$$\alpha+\beta\le 1;\ s\in[0,1]$$
$$\alpha\in[0,1],\beta\in[0,1]$$
$$i=1,2,......,p;\ j=1,2,......,m$$

Step-5: Solve the above crisp model (8) by using appropriate mathematical programming algorithm to get optimal solution of objective function.
Step-6: Stop.

V. SOLUTION OF MULTI-OBJECTIVE STRUCTURAL OPTIMIZATION PROBLEM BY FUZZY AND INTUITIONISTIC FUZZY OPTIMIZATION TECHNIQUE

The multi-objective structural model (1) can be expressed as parametric intuitionistic form as

$$\text{Minimize } WT(A;s) \tag{9}$$

$$\text{Minimize } \delta(A;s)$$

$$\text{subject to } \sigma(A;s)\le([\sigma];s)$$

$$A^{\min}\le A\le A^{\max}$$

Where $A=(A_1,A_2,....,A_n)^T$
To solve the MOSOP (9) step 1 of 4.5is used. After that according to step 2 pay-off matrix is formulated

$$\begin{array}{cc} & \begin{array}{cc} WT(A;s) & \delta(A;s) \end{array} \\ \begin{array}{c} A^1 \\ A^2 \end{array} & \begin{bmatrix} WT^*\left(A^1;s\right) & \delta\left(A^1;s\right) \\ WT\left(A^2;s\right) & \delta^*\left(A^2;s\right) \end{bmatrix} \end{array}$$

In next step following step 2 we calculate the bound of the objective U_1^{Acc},L_1^{Acc} and $U_1^{\mathrm{Re}j},L_1^{\mathrm{Re}j}$ for weight function $WT(A;s)$,such that $L_1^{Acc}<WT(A;s)<U_1^{Acc}$ and $L_2^{\mathrm{Re}j}<WT(A;s)<U_2^{\mathrm{Re}j}$ and $U_2^{Acc},L_2^{Acc};U_2^{\mathrm{Re}j},L_2^{\mathrm{Re}j}$ for deflection $\delta(A;s)$, such that $L_2^{Acc}<WT(A;s)<U_2^{Acc}$ and $L_2^{\mathrm{Re}j}<\delta(A;s)<U_2^{\mathrm{Re}j}$ with the condition $U_i^{Acc}=U_i^{\mathrm{Re}j};\ L_i^{\mathrm{Re}j}=L_i^{Acc}+\varepsilon_i\quad for\ i=1,2$ so as $0<\varepsilon_i<\left(U_i^{Acc}-L_i^{Acc}\right)$ are identified.
According to IFO technique considering membership and non-membership function for MOSOP (9)

$$\mu_{WT(A;s)}\left(WT(A;s)\right)=\begin{cases} 1 & if\ \ WT(A;s)\le L_{WT}^{Acc} \\ \dfrac{e^{-T\left(\frac{WT(A;s)-L_{WT}^{Acc}}{U_{WT}^{Acc}-L_{WT}^{Acc}}\right)}-e^{-T}}{1-e^{-T}} & if\ \ L_{WT}^{Acc}\le WT(A;s)\le U_{WT}^{Acc} \\ 0 & if\ \ WT(A;s)\ge U_{WT}^{Acc} \end{cases}$$

$$\upsilon_{WT(A;s)}\left(WT(A;s)\right)=\begin{cases} 0 & if\ \ WT(A;s)\le L_{WT}^{\mathrm{Re}j} \\ \left(\dfrac{WT(A;s)-L_{WT}^{\mathrm{Re}j}}{U_{WT}^{\mathrm{Re}j}-L_{WT}^{\mathrm{Re}j}}\right)^2 & if\ \ L_{WT}^{\mathrm{Re}j}\le WT(A;s)\le U_{WT}^{\mathrm{Re}j} \\ 1 & if\ \ WT(A;s)\ge U_{WT}^{\mathrm{Re}j} \end{cases}$$

and

$$\mu_{\delta(A;s)}\left(\delta(A;s)\right)=\begin{cases} 1 & if \quad \delta(A;s)\le L_{\delta}^{Acc} \\ \dfrac{e^{-T\left(\frac{\delta(A;s)-L_{\delta}^{Acc}}{U_{\delta}^{Acc}-L_{\delta}^{Acc}}\right)}-e^{-T}}{1-e^{-T}} & if \quad L_{\delta}^{Acc}\le \delta(A;s)\le U_{\delta}^{Acc} \\ 0 & if \quad \delta(A;s)\ge U_{\delta}^{Acc} \end{cases}$$

$$\upsilon_{\delta(A;s)}\left(\delta(A;s)\right)=\begin{cases} 0 & if \quad \delta(A;s)\le L_{\delta}^{\mathrm{Re}\,j} \\ \left(\dfrac{\delta(A;s)-L_{\delta}^{\mathrm{Re}\,j}}{U_{\delta}^{\mathrm{Re}\,j}-L_{\delta}^{\mathrm{Re}\,j}}\right)^2 & if \quad L_{\delta}^{\mathrm{Re}\,j}\le \delta(A;s)\le U_{\delta}^{\mathrm{Re}\,j} \\ 1 & if \quad \delta(A;s)\ge U_{\delta}^{\mathrm{Re}\,j} \end{cases}$$

crisp non-linear programming problem is formulated as follows

$$Max\left(Min\left(\mu_{WT}\left(WT\left(A;s\right)\right)\right),\mu_{\delta}\left(\delta\left(A;s\right)\right)\right)-Min\left(Max\left(\upsilon_{WT}\left(WT\left(A;s\right)\right)\right),\upsilon_{\delta}\left(\delta\left(A;s\right)\right)\right) \tag{10}$$

$$\begin{aligned}
&subject\ to\\
&\mu_{WT}\left(WT\left(A;s\right)\right)+\upsilon_{WT}\left(WT\left(A;s\right)\right)<1;\ \mu_{\delta}\left(\delta\left(A;s\right)\right)+\upsilon_{\delta}\left(\delta\left(A;s\right)\right)<1;\\
&\mu_{WT}\left(WT\left(A;s\right)\right)>\upsilon_{WT}\left(WT\left(A;s\right)\right);\ \mu_{\delta}\left(\delta\left(A;s\right)\right)>\upsilon_{\delta}\left(\delta\left(A;s\right)\right);\\
&\mu_{WT}\left(WT\left(A;s\right)\right)\ge 0,\upsilon_{WT}\left(WT\left(A;s\right)\right)\ge 0;\ \mu_{\delta}\left(\delta\left(A;s\right)\right)\ge 0,\upsilon_{\delta}\left(\delta\left(A;s\right)\right)\ge 0;\\
&\sigma\left(A;s\right)\le\left(\left[\sigma\right];s\right);A>0;\ s\in\left[0,1\right]
\end{aligned}$$

According to Angelov[2], the above problem can be written as

$$Maximize\ \left(\alpha-\beta\right) \tag{11}$$

$$\begin{aligned}
&subject\ to\\
&\mu_{WT}\left(WT\left(A;s\right)\right)\ge\alpha;\ \upsilon_{WT}\left(WT\left(A;s\right)\right)\le\beta;\\
&\mu_{\delta}\left(\delta\left(A;s\right)\right)\ge\alpha;\ \upsilon_{\delta}\left(\delta\left(A;s\right)\right)\le\beta;\\
&\sigma\left(A;s\right)\le\left(\left[\sigma\right];s\right),\ \alpha+\beta\le 1;\\
&A>0,\alpha\in\left[0,1\right],\beta\in\left[0,1\right],\ s\in\left[0,1\right]
\end{aligned}$$

Solve the above crisp model (11) by an appropriate mathematical programming algorithm to get optimal solution and hence objective function i.e structural weight and deflection of loaded joint will get the Pareto optimal solution.

VI. NUMERICAL ILLUSTRATION

A well known three bar planer truss is considered is to minimize weight $WT\left(A_1,A_2\right)$ of the structure and minimize the deflection $\delta\left(A_1,A_2\right)$ at a loading point of a statistically loaded three bar planer truss subject to stress constraints on each of the truss members

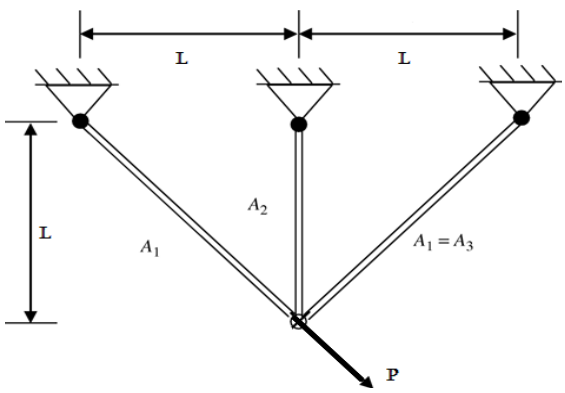

Fig.1. Design of Three Bar Planer Truss

The multi-objective optimization problem can be stated as follows

$$Minimize \quad WT(A_1, A_2) = \rho L\left(2\sqrt{2}A_1 + A_2\right) \tag{12}$$

$$Minimize \quad \delta(A_1, A_2) = \frac{PL}{E\left(A_1 + \sqrt{2}A_2\right)}$$

$$subject\ to\ \sigma_1(A_1, A_2) = \frac{P\left(\sqrt{2}A_1 + A_2\right)}{\left(2A_1^2 + 2A_1A_2\right)} \le \left[\sigma_1^T\right];$$

$$\sigma_2(A_1, A_2) = \frac{P}{\left(A_1 + \sqrt{2}A_2\right)} \le \left[\sigma_2^T\right];$$

$$\sigma_3(A_1, A_2) = \frac{PA_2}{\left(2A_1^2 + 2A_1A_2\right)} \le \left[\sigma_3^C\right];$$

$$A_i^{\min} \le A_i \le A_i^{\max} \quad i = 1, 2$$

Where applied load $\tilde{P}^i = \left(\left(19, 20, 21; w_p\right)\left(18, 20, 22; \tau_p\right)\right)$; material density $\tilde{\rho}^i = \left(\left(99, 100, 101; w_\rho\right)\left(98, 100, 102; \tau_\rho\right)\right)$;length $L = 1m$;Young's modulus $E = 2\times10^8$; A_1 = Cross section of bar-1 and bar-3; A_2 = Cross section of bar-2; δ is deflection of loaded joint. $\left[\sigma_1^T\right] = \left(\left(19.5, 20, 20.5; w_{\sigma_1^T}\right)\left(18, 20, 21; \tau_{\sigma_1^T}\right)\right)$ and $\left[\sigma_2^T\right] = \left(\left(18.5, 20, 20.5; w_{\sigma_2^T}\right)\left(18, 20, 21; \tau_{\sigma_2^T}\right)\right)$ are maximum allowable tensile stress for bar 1 and bar 2 respectively, $\left[\sigma_3^C\right] = \left(\left(14, 15, 16; w_{\sigma_3^C}\right)\left(13, 15, 17; \tau_{\sigma_3^C}\right)\right)$ is maximum allowable compressive stress for bar 3.

Now parameterized value of interval valued function can be calculated as

$$\hat{P} = \left(\left(19.5 + \frac{0.25}{w_p} + \frac{0.5}{\tau_p}\right)^{1-s}\left(20.5 + \frac{.25}{w_p} + \frac{0.5}{\tau_p}\right)^s\right);\ \hat{\rho} = \left(\left(99.5 + \frac{.125}{w_\rho} + \frac{0.5}{\tau_\rho}\right)^{1-s}\left(100.5 + \frac{.125}{w_\rho} + \frac{0.5}{\tau_\rho}\right)^s\right);$$

$$\hat{\sigma}_1^T = \left(\left(19.75 + \frac{0.125}{w_{\sigma_1^T}} + \frac{0.5}{\tau_{\sigma_1^T}}\right)^{1-s}\left(20.25 + \frac{0.125}{w_{\sigma_1^T}} + \frac{0.25}{\tau_{\sigma_1^T}}\right)^s\right);$$

$$\hat{\sigma}_2^T = \left(\left(19.25 + \frac{0.375}{w_{\sigma_2^T}} + \frac{0.5}{\tau_{\sigma_2^T}}\right)^{1-s}\left(20.25 + \frac{0.125}{w_{\sigma_2^T}} + \frac{0.25}{\tau_{\sigma_2^T}}\right)^s\right);$$

$$\hat{\sigma}_3^C = \left(\left(14.5 + \frac{0.25}{w_{\sigma_3^C}} + \frac{.5}{\tau_{\sigma_3^C}}\right)^{1-s}\left(15.5 + \frac{0.25}{w_{\sigma_3^C}} + \frac{0.5}{\tau_{\sigma_3^C}}\right)^s\right);$$

Table.1 The pay-off matrices for different values of w,τ pessimistic value of s

The pessimistic value of s=0.2			
$w_{\rho} = w_{\rho} = w_{\sigma_2^T} = w_{\sigma_3^C} = w$ $\tau_{\rho} = \tau_{\rho} = \tau_{\sigma_2^T} = \tau_{\sigma_3^C} = \tau$	$w = 0.3, \tau = 0.6$	$w = 0.5, \tau = 0.5$	$w = 0.7, \tau = 0.2$
Pay-off matrices	$\begin{array}{cc} & WT(A_1,A_2) \quad \delta(A_1,A_2) \\ \begin{array}{c} A^1 \\ A^2 \end{array} & \begin{bmatrix} 2.114347 & 21.33333 \\ 19.14214 & 1.769768 \end{bmatrix} \end{array}$ $U_{WT}^{\upsilon} = U_{WT}^{\mu} = 19.14214,$ $L_{WT}^{\upsilon} = L_{WT}^{\mu} + \varepsilon_1 = 2.114347 + \varepsilon_1;$ $0 < \varepsilon_1 < (19.14214 - 2.114347)$ $U_{\delta}^{\upsilon} = U_{\delta}^{\mu} = 21.33333,$ $L_{\delta}^{\upsilon} = L_{\delta}^{\mu} + \varepsilon_2 = 1.769768 + \varepsilon_2;$ $0 < \varepsilon_2 < (21.33333 - 1.769768)$	$\begin{array}{cc} & WT(A_1,A_2) \quad \delta(A_1,A_2) \\ \begin{array}{c} A^1 \\ A^2 \end{array} & \begin{bmatrix} 2.104498 & 21.04976 \\ 19.14214 & 1.755959 \end{bmatrix} \end{array}$ $U_{WT}^{\upsilon} = U_{WT}^{\mu} = 19.14214,$ $L_{WT}^{\upsilon} = L_{WT}^{\mu} + \varepsilon_1 = 2.104498 + \varepsilon_1;$ $0 < \varepsilon_1 < (19.14214 - 2.104498)$ $U_{\delta}^{\upsilon} = U_{\delta}^{\mu} = 21.04976,$ $L_{\delta}^{\upsilon} = L_{\delta}^{\mu} + \varepsilon_2 = 1.755959 + \varepsilon_2;$ $0 < \varepsilon_2 < (21.04976 - 1.755959)$	$\begin{array}{cc} & WT(A_1,A_2) \quad \delta(A_1,A_2) \\ \begin{array}{c} A^1 \\ A^2 \end{array} & \begin{bmatrix} 2.114347 & 21.33333 \\ 19.14214 & 1.769768 \end{bmatrix} \end{array}$ $U_{WT}^{\upsilon} = U_{WT}^{\mu} = 19.14214,$ $L_{WT}^{\upsilon} = L_{WT}^{\mu} + \varepsilon_1 = 2.114347 + \varepsilon_1;$ $0 < \varepsilon_1 < (19.14214 - 2.114347)$ $U_{\delta}^{\upsilon} = U_{\delta}^{\mu} = 21.3333,$ $L_{\delta}^{\upsilon} = L_{\delta}^{\mu} + \varepsilon_2 = 1.769768 + \varepsilon_2;$ $0 < \varepsilon_2 < (21.33333 - 1.769768)$

Table.2 The pay-off matrices for different values of w,τ moderate value of s

The moderate value of s=0.5			
$w_{\rho} = w_{\rho} = w_{\sigma_2^T} = w_{\sigma_3^C} = w$ $\tau_{\rho} = \tau_{\rho} = \tau_{\sigma_2^T} = \tau_{\sigma_3^C} = \tau$	$w = 0.3, \tau = 0.6$	$w = 0.5, \tau = 0.5$	$w = 0.7, \tau = 0.2$
Pay-off matrices	$\begin{array}{cc} & WT(A_1,A_2) \quad \delta(A_1,A_2) \\ \begin{array}{c} A^1 \\ A^2 \end{array} & \begin{bmatrix} 2.114347 & 21.33333 \\ 19.14214 & 1.769768 \end{bmatrix} \end{array}$ $U_{WT}^{\upsilon} = U_{WT}^{\mu} = 19.14214,$ $L_{WT}^{\upsilon} = L_{WT}^{\mu} + \varepsilon_1 = 2.114347 + \varepsilon_1;$ $0 < \varepsilon_1 < (19.14214 - 2.114347)$ $U_{\delta}^{\upsilon} = U_{\delta}^{\mu} = 21.3333,$ $L_{\delta}^{\upsilon} = L_{\delta}^{\mu} + \varepsilon_2 = 1.769768 + \varepsilon_2;$ $0 < \varepsilon_2 < (21.33333 - 1.769768)$	$\begin{array}{cc} & WT(A_1,A_2) \quad \delta(A_1,A_2) \\ \begin{array}{c} A^1 \\ A^2 \end{array} & \begin{bmatrix} 2.132633 & 21.12463 \\ 19.14214 & 1.780637 \end{bmatrix} \end{array}$ $U_{WT}^{\upsilon} = U_{WT}^{\mu} = 19.14214,$ $L_{WT}^{\upsilon} = L_{WT}^{\mu} + \varepsilon_1 = 2.132633 + \varepsilon_1;$ $0 < \varepsilon_1 < (19.14214 - 2.132633)$ $U_{\delta}^{\upsilon} = U_{\delta}^{\mu} = 21.12463,$ $L_{\delta}^{\upsilon} = L_{\delta}^{\mu} + \varepsilon_2 = 1.780637 + \varepsilon_2;$ $0 < \varepsilon_2 < (21.12463 - 1.780637)$	$\begin{array}{cc} & WT(A_1,A_2) \quad \delta(A_1,A_2) \\ \begin{array}{c} A^1 \\ A^2 \end{array} & \begin{bmatrix} 2.160705 & 22.10642 \\ 19.14214 & 1.893095 \end{bmatrix} \end{array}$ $U_{WT}^{\upsilon} = U_{WT}^{\mu} = 19.14214,$ $L_{WT}^{\upsilon} = L_{WT}^{\mu} + \varepsilon_1 = 2.160705 + \varepsilon_1;$ $0 < \varepsilon_1 < (19.14214 - 2.160705)$ $U_{\delta}^{\upsilon} = U_{\delta}^{\mu} = 22.10642,$ $L_{\delta}^{\upsilon} = L_{\delta}^{\mu} + \varepsilon_2 = 1.893095 + \varepsilon_2;$ $0 < \varepsilon_2 < (22.10642 - 1.893095)$

Table.3 The pay-off matrices for different values of w,τ optimistic value of s

The optimistic value of s=0.8			
$w_{\rho} = w_{\rho} = w_{\sigma_2^T} = w_{\sigma_3^C} = w$ $\tau_{\rho} = \tau_{\rho} = \tau_{\sigma_2^T} = \tau_{\sigma_3^C} = \tau$	$w = 0.3, \tau = 0.6$	$w = 0.5, \tau = 0.5$	$w = 0.7, \tau = 0.2$
Pay-off matrices	$\begin{array}{cc} & WT(A_1,A_2) \quad \delta(A_1,A_2) \\ \begin{array}{c} A^1 \\ A^2 \end{array} & \begin{bmatrix} 2.169528 & 21.33333 \\ 19.14214 & 1.819471 \end{bmatrix} \end{array}$ $U_{WT}^{\upsilon} = U_{WT}^{\mu} = 19.14214,$ $L_{WT}^{\upsilon} = L_{WT}^{\mu} + \varepsilon_1 = 2.169528 + \varepsilon_1;$ $0 < \varepsilon_1 < (19.14214 - 2.169528)$ $U_{\delta}^{\upsilon} = U_{\delta}^{\mu} = 21.33333,$ $L_{\delta}^{\upsilon} = L_{\delta}^{\mu} + \varepsilon_2 = 1.819471 + \varepsilon_2;$ $0 < \varepsilon_2 < (21.33333 - 1.819471)$	$\begin{array}{cc} & WT(A_1,A_2) \quad \delta(A_1,A_2) \\ \begin{array}{c} A^1 \\ A^2 \end{array} & \begin{bmatrix} 2.161155 & 21.19976 \\ 19.14214 & 1.805661 \end{bmatrix} \end{array}$ $U_{WT}^{\upsilon} = U_{WT}^{\mu} = 19.14214,$ $L_{WT}^{\upsilon} = L_{WT}^{\mu} + \varepsilon_1 = 2.161155 + \varepsilon_1;$ $0 < \varepsilon_1 < (19.14214 - 2.161155)$ $U_{\delta}^{\upsilon} = U_{\delta}^{\mu} = 21.19976,$ $L_{\delta}^{\upsilon} = L_{\delta}^{\mu} + \varepsilon_2 = 1.805661 + \varepsilon_2;$ $0 < \varepsilon_2 < (21.19976 - 1.805661)$	$\begin{array}{cc} & WT(A_1,A_2) \quad \delta(A_1,A_2) \\ \begin{array}{c} A^1 \\ A^2 \end{array} & \begin{bmatrix} 2.209459 & 21.99954 \\ 19.14214 & 1.918109 \end{bmatrix} \end{array}$ $U_{WT}^{\upsilon} = U_{WT}^{\mu} = 19.14214,$ $L_{WT}^{\upsilon} = L_{WT}^{\mu} + \varepsilon_1 = 2.209459 + \varepsilon_1;$ $0 < \varepsilon_1 < (19.14214 - 2.209459)$ $U_{\delta}^{\upsilon} = U_{\delta}^{\mu} = 21.99954,$ $L_{\delta}^{\upsilon} = L_{\delta}^{\mu} + \varepsilon_2 = 1.918109 + \varepsilon_2;$ $0 < \varepsilon_2 < (21.99954 - 1.918109)$

Now using membership and non membership function for $T=2$ intuitionistic optimization problem can be formulatedas similar as (11) and solving these optimal design variables and objective functions for different values of s,w,σ can be obtained as follows.

Table.4 The optimum values of design variables for different values of s,w,τ

The pessimistic value of s=0.2			
Value of $\varepsilon_1,\varepsilon_2$, $w_\rho = w_\rho = w_{\sigma_2^T} = w_{\sigma_3^C} = w$ $\tau_\rho = \tau_\rho = \tau_{\sigma_2^T} = \tau_{\sigma_3^C} = \tau$	$w=0.3,\tau=0.6$ $\varepsilon_1=1.68,\varepsilon_2=2.19$	$w=0.5,\sigma=0.5$ $\varepsilon_1=1.68,\varepsilon_2=2.11$	$w=0.7,\tau=0.2$ $\varepsilon_1=1.65,\varepsilon_2=2.37$
$A_1\times10^{-4}m^2$	0.5463992	0.5514346	0.5433765
$A_2\times10^{-4}m^2$	2.805193	2.953187	2.980011
$WT\times10^2 KN$	4.350643	4.512880	4.516911
$\delta\times10^{-7}m$	4.733089	4.483264	4.740406

Table.5 The optimum values of design variables for different values of s,w,τ

The moderate value of s=0.5			
Value of $\varepsilon_1,\varepsilon_2$, $w_\rho = w_\rho = w_{\sigma_2^T} = w_{\sigma_3^C} = w$ $\tau_\rho = \tau_\rho = \tau_{\sigma_2^T} = \tau_{\sigma_3^C} = \tau$	$w=0.3,\tau=0.6$ $\varepsilon_1=1.67,\varepsilon_2=2.29$	$w=0.5,\tau=0.5$ $\varepsilon_1=1.67,\varepsilon_2=2.21$	$w=0.7,\tau=0.2$ $\varepsilon_1=1.65,\varepsilon_2=2.47$
$A_1\times10^{-4}m^2$	0.5489754	0.5500707	0.5514305
$A_2\times10^{-4}m^2$	2.979649	2.979080	3.006944
$WT\times10^2 KN$	4.532386	4.534915	4.566626
$\delta\times10^{-7}m$	4.547900	4.512622	4.756908

Table.6 The optimum values of design variables for different values of s,w,τ

The optimistic value of s=0.8			
Value of $\varepsilon_1,\varepsilon_2$, $w_\rho = w_\rho = w_{\sigma_2^T} = w_{\sigma_3^C} = w$ $\tau_\rho = \tau_\rho = \tau_{\sigma_2^T} = \tau_{\sigma_3^C} = \tau$	$w=0.3,\tau=0.6$ $\varepsilon_1=1.67,\varepsilon_2=2.4$	$w=0.5,\tau=0.5$ $\varepsilon_1=1.67,\varepsilon_2=2.32$	$w=0.7,\tau=0.2$ $\varepsilon_1=1.65,\varepsilon_2=2.6$
$A_1\times10^{-4}m^2$	0.5549861	0.5523943	0.5649746
$A_2\times10^{-4}m^2$	2.999530	2.998970	3.025163
$WT\times10^2 KN$	4.569267	4.561377	4.623152
$\delta\times10^{-7}m$	4.578514	4.546971	4.780645

Here we get solution for different tolerance ε_1 and ε_2 ,non-linear membership of IFO method and for different values of w,τ and s .

VII. CONCLUSION

In this paper, an IFO approach has been used to solve IFNLPP with IF resources as well as IF coefficients which may be considered as triangular intuitionistic fuzzy numbers. The proposed method is utilized with non-linear membership function to a structural design . In this test problem we have considered a three bar truss design where weight of the structure and deflection of loaded joint is to be minimized. At first step triangular intuitionistic fuzzy number has been transformed into an interval number by interval approximation method. After that parametric interval valued functional form are created from that interval numbers so that the solution of the undertaken numerical problem can be solved by intuitionistic fuzzy optimization method given by Angelov [2]. As it has been seen that there is a few method in literature for solving IFNLP and the proposed method has been applied to a fully IFNLP problem so the method will be very beneficial and applicable for solving NLPP arising in other field of engineering in intuitionistic fuzzy environment.

ACKNOWLEDGMENT

Authors would like to thank referees for their helpful comments.

REFERENCES

[1] D.Dubey, andA. Mehra, Linear programming with triangular intuitionistic fuzzy number. In *Proceedings of the 7th conference of the European Society for Fuzzy Logic and Technology* (pp. 563-569). Atlantis Press,2011.

[2] P.P.Angelov, Optimization in intuitionistic fuzzy environment. *Fuzzy Sets and Systems* 86, 1997,pp-299–306.

[3] S.Dey, and T.K.Roy, Optimized solution of two bar truss design using intuitionistic fuzzy optimization technique , *International Journal of Information Engineering and Electronic Business*,2014(3),2014,pp- 45-51.

[4] K. Atanassov, Intuitionistic fuzzy sets, *Fuzzy sets and Systems*, 20,1986,pp-87-96.

[5] L.A.Zadeh, Fuzzy set, *Information and Control*, vol.8, no.3,1965, pp.338-353.

[6] H.J.Zimmermann, Fuzzy programming and linear programming with several objective functions. *Fuzzy sets and systems*, *1*(1), 1978.pp-45-55.

[7] S.K.Singh, and S.P.Yadav, Intuitionistic fuzzy non linear programming problem: Modeling and optimization in manufacturing systems.*Journal of Intelligent & Fuzzy Systems*, *28*(3),2015,pp-1421-1433.