Comparative study of Gradient based image denoising methods

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*Abstract:***Images are playing very key role in various applications like medical imaging, remote sensing, and scientificresearch and in all these applications image clarity plays a major role. Any image that is captured has to be processed for noise removal because noise in image is unavoidable and mixes with image either during capturing or during transmission. Various image denoising methods are developed earlier and are also working for better one. Among various methods developed Sparsity based Non local means methods are giving better denoising results. Main drawbacks of image denoising methods are they smoothens the fine details of the denoised image. This drawback can be overcome by using gradient histogram as reference while restoring the image. This paper discuss about some of gradient based image denoising methods and their comparative results.**

Keywords:NCSR, PSNR, SSIM, FSIM, Gradient Histogram

1. INTRODUCTION

Image denoising plays important role in image processing applications. Natural image statistics plays an important role inimage denoising, and various natural image priors, includinggradient based, sparse representation based [3] and nonlocal self-similaritybased ones [4], have been widely studied and exploited for noise removal. No matter how good cameras are, an image improvement is always desirable to extend their range of action. Noise is random variations of intensities in image and appears like grains in image. An image is corrupted by noise during its transmission. There are many types of noises that can be seen in an image like additive white gaussian noise (AWGN), poission noise, salt and pepper noise, shot noise etc. Denoising is process of recovering latent clean image from corrupted noisy image. But image denoising introduces artifacts and blur in denoised images. The main goal of image denoising is to reconstruct an image that is as close as possible to original image. There are many denoising algorithms that are developed and these algorithms depend on the type of noise the image is corrupted with. Denoising of image with additive white gaussian noise is easy when compared to other multiplicative noises. The algorithms discussed in this paper are for AWGN. Denoising first started with filtering methods, [2], and later various other methods are developed. Studies about natural image statistics revealed the fact that image gradientsare crucial for perception and analysis of natural images. These studies helped in developing gradient based image denoising methods.

The rest of paper is organized as follows. Section I describes the concepts of total variation regularization, Sparsity and Nonlocal means methods. Section II describes the concepts of surveyed gradient based image denoising methods and various performance measures. Section III provides experimental results,comparisons and conclusion.

Section I

1.1 Total variation regularization

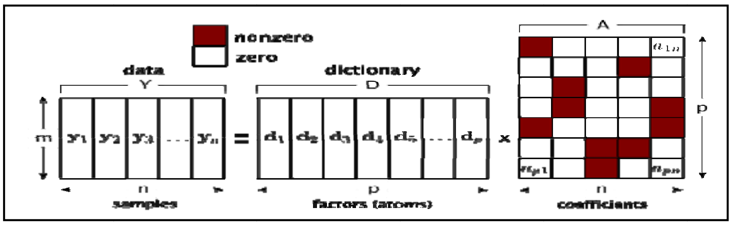
This total variation regularization method is first gradient based denoising method developed by Rudin, Osher and Fatemi [1]. This method overcomes the drawback of earlier filter methods which smoothens the edges to greater extent. This method is developed on based on concept that the integral of absolute gradient of image is high.When noise is added to image, the image contains excessive and spurious details and such images have high total variation. The total variance that is present in image is reduced with some constraints. This removes unwanted details such as noise from image while preserving important details such as edges. The constraints that are imposed depend on noise statistics.

$$\int_{\Omega}(u(x) - v(x))dx = 0 \text{ And } \int_{\Omega}(u(x) - v(x))dx = \sigma^2$$

Here u(x) is original image and v(x) is noisy image. σis known standard deviation of gaussian white noise. This method smoothens the fine details that are present in image.

Sparsity

Most of signals and natural images can be compactly represented as linear combination of dictionary atoms. This type of representation is called as sparse representation.



Based on this Sparsity concept image denoising can be done, sparsity based methods comprise of two steps, 1) sparse coding 2) dictionary update

1. Sparse coding:

Given an image and dictionary, sparse coefficients are to be computed. Selection of dictionary is very important step in sparse coding. A variety of dictionaries were developed in response to the rising need. The newly developed dictionaries emerged from one of two sources — either a mathematical model of the data, or a set of realizations of the data. Dictionaries of the first type are characterized by an analytic formulation and a fast implicit implementation, while dictionaries of the second type deliver increased flexibility and the ability to adapt to specific signal data. Earlier dictionary is formed using wavelet coefficients of image. After adaptive dictionaries are developed which gave better results when compared to previous fixed dictionaries. In adaptive dictionary learning, the dictionary is adaptively selected based on denoising image which is reason for giving better results. But these adaptive dictionaries are unstable so PCA sub dictionaries are developed. In this k PCA sub dictionaries are formed and while sparse coding one among these k PCA sub dictionaries [12] are selected which enables more sparse representation of image.

$$\alpha_x = \underset{x}{\operatorname{argmin}} \|x - D\alpha\|_2^2$$

2. Dictionary update:

Based on sparse coefficients α_x, and the image x, the dictionary D has to be updated for better representation of image. The desire to efficiently train a generic dictionary for sparse signal representation led Aharon, Elad and Bruckstein to develop the K-SVD algorithm in 2005 [14]. The main contribution of the K-SVD is that the dictionary update is performed atom-by-atom in a simple and efficient process. The K-SVD algorithm takes its name from the Singular- Value-Decomposition (SVD) process that forms the core of the atom update step and which is repeated K times, as the number of atoms.

$$\mathrm{D}^{(n+1)} = \mathrm{D}^{(n)} - \eta \sum_{i=1}^{N} \left(\mathrm{D}^{(n)} x_i - y_i\right) x_i^{T}$$

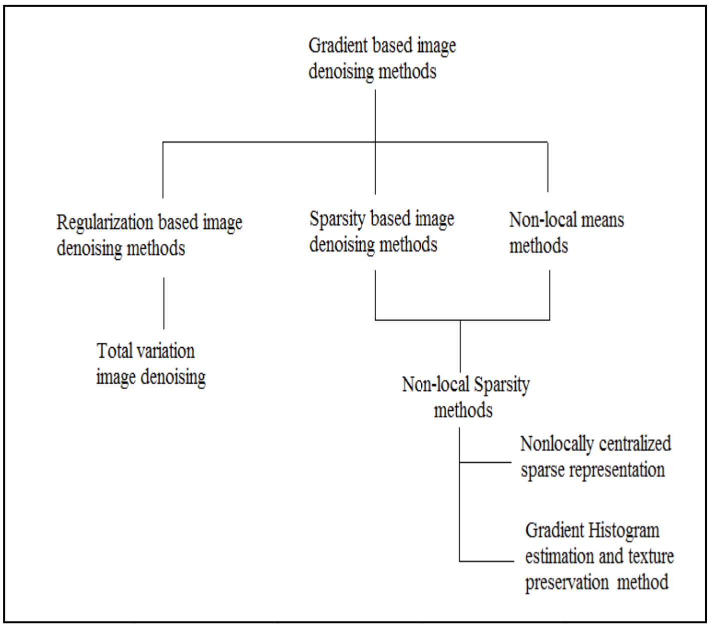


Fig 1.1 flow chart for various gradient based image denoising methods

1.2 Sparsity based method

Image Denoising via Sparse and Redundant Representations over Learned Dictionaries [8] was proposed by Michael Elad and Michal Aharon. Sparsity based methods using leaned dictionaries gave better results when compared to other earlier methods where fixed dictionaries are used such as wavelets, Curvelets etc. in this they proposed an algorithm where the dictionary is trained from the corrupted image itself instead of database images. This is a patch based method, where the noisy image is divided into patches and each patch is sparse codded based on over-complete dictionary. The algorithm used for this purpose is K-means Singular Value Decomposition with Orthogonal Matching Pursuit (K-SVD OMP). There are two stages for accomplishing this task. Stage one is sparse coding where sparse coefficients are computed for each patch based on dictionary and this is done using OMP method. Stage two is dictionary update based on the above obtained sparse coefficients the dictionary is updated one column at a time by minimizing MSE and this is done using K-SVD method. Each patch is overlapped to obtain final denoised image.

1.3 Non local means method

A non-local algorithm for image denoising [4] was developed by A. Buades, B. Coll, and J. Morel. This method is developed based on non-local similarities that are abundantly present in natural images. Natural images also have enough redundancy to be restored by NL-means. This method is developed to over-come the inability of sparse coding techniques to handle similar patches where similar patches sometimes admit very different estimates. In non-local means method the nonlocal mean of every pixel is calculated as weighted average of all pixels in image. This weight depends on similarity between pixels. The similarity between two pixels depends on the similarity of the intensity gray level vectors. The similarity is measured as decreasing function of weighted Euclidean distance. The pixels with similar gray level with neighborhood have larger weights in average. The non-local means approach to image restoration explicitly exploits self-similarities in natural images to average out the noise among similar patches. The drawback of this method is it can handle cases where the noisy image does not have similar patches.

1.4 Non local sparse method

A Non-local Sparse Model for Image Restoration [5] was developed by J. Mairal. Non-local means filtering is proven to be very effective in general, but it fails in some cases. In the extreme, when a patch does not look like any other one in the image, it is impossible to exploit self-similarities to denoise the corresponding pixel value.Sparse image models can handle such situations by exploiting the redundancy between overlapping patches, but they suffer from another drawback: Similar patches sometimes admit very different estimates due to the potential instability of sparse decompositions which can result in practice in noticeable reconstruction artifacts. In this method both Sparsity and nonlocal means methods are combined to over-come the drawbacks of both methods. These two sparse coding and nonlocal means methods are combined using simultaneous sparse coding technique. In this nonlocal sparse model the image is made into patches and all the similar patches are made into a set and this set is sparsely decomposed using same dictionary forcing similar patches to have similar estimate values.

Section II

2.1 Centralized nonlocal sparse method

Non-locally Centralized Sparse Representation for Image Restoration [6] was developed by Weisheng Dong. Sparse representation models code an image patch as a linear combination of a few atoms chosen out from an over-complete dictionary and shows promising results for image restoration. However, due to the degradation of the observed image (e.g., noisy, blurred, and/or down-sampled), the sparse representations by conventional models may not be accurate enough for a faithful reconstruction of the original image.

For an observed image y, the problem of image restoration (IR) can be generally formulated by

$$Y = Hx + v$$

Mathematically, the sparse representation model assumes that a signal x can be represented as $x \approx \emptyset\alpha$,where $\emptyset$ is an over complete dictionary and most entries of the coding vector α are zero or close to zero.The sparse decomposition of x can be obtained by solving

$$\alpha_x = \arg\min_\alpha \|\alpha\|_0, \mathrm{s.t.} \|x - \emptyset\alpha\|_2 \leq \epsilon$$

Where $\|\cdot\|_0$ is a pseudo norm that counts the number of non-zero entries in α,and ε is a small constant controlling the approximation error.Since the aboe formulation is an NP hard combinatorial optimization problem, it is often relaxed to the convex ℓ1-minimization. The ℓ1-norm based sparse coding problem can be generally formulated in the following Lagrangian form:

$$\alpha_x = \arg\min_\alpha\{\|x - \emptyset\alpha\|_2^2 + \lambda\|\alpha\|_1\}$$

where constant λ denotes the regularization parameter. With an appropriate selection of the regularization parameter λ, we can get a good balance between the sparse approximation error of x and the sparsity of α.

In the scenario of image denoising, what we observed is the degraded image signal y via y = Hx + υ. Earlier various sparse models are developed for image denoising, later various studies revealed that further development of these methods can given better results. Further developemnt of earlier nonlacal sparse models is NCSR nonlocal sparse model, which is improved by the concept of sparse coding noise. In real time application original image is not available so by using noisy image only the original image has to be restored using nonlocal sparse model. Sparse coding noise is defined as the difference between sparse coefficient of noisy image and good estimate of original image. By reducing the difference we can improve the quality of restoring image. A good estimate of the sparse coding coefficients of the original image is obtained by exploiting the nonlocal self-similarities of natural images

To recover x from y, first y is sparsely coded with respect to $\emptyset$ by solving the following minimization problem:

$$\alpha_y = \arg\min_{\alpha}\left\{\|y - H\emptyset \times \alpha\|_2^2 + \lambda\sum_i \|\alpha_i - \beta_i\|_p\right\}$$

The sparse coding model can be solved iteratively.We first initialize β_i as 0 i.e., and solve for the sparse coding vector, denoted by $\alpha_y^{(0)}$, using some standard sparse coding algorithm. Then we can get the initial estimation of x, denoted by $x^{(0)}$ via $x^{(0)} = \emptyset \times \alpha_y^{(0)}$. Based on $x^{(0)}$, we search for the similar patches to each patch i , and hence the nonlocal estimate of β_i is updated. The updated estimation of α_x, denoted by $\beta_i^{(0)}$, will then be used to improve the accuracy of the sparse codes and thus improve the IR quality. Such a procedure is iterated until convergence.In the lth iteration, the sparse vector is obtained by solving the following minimization problem

$$\alpha_y^{(l)} = \arg\min_{\alpha}\left\{\|y - H\emptyset \times \alpha\|_2^2 + \lambda\sum_i \left\|\alpha_i - \beta_i^{(l)}\right\|_p\right\}$$

The restored image is then updated as $\hat{x}^{(l)} = \emptyset \times \alpha_y^{(l)}$.

2.2 Gradient histogram based image restoration method

This method Gradient Histogram Estimation and Preservation for Texture Enhanced Image Denoising [7] was proposed by Wangmeng Zuo, Lei Zhang, Chunwei Song, David Zhang and Huijun Gao. Image denoising, which aims to estimate the latent clean image x from its noisy observation y, is a classical yet still active topic in image processing and low level vision. One widely used data observation model is y = x + v, where v is additive white Gaussian noise (AWGN).One popular approach to image denoising is the variational method, in which the denoised image is obtained by

$$\hat{x} = \arg\min_{x}\left\{\frac{1}{2\sigma^2}\|y - x\|^2 + \lambda \cdot R(x)\right\}$$

Where R(x) denotes some regularization term and λ is a positive constant. The specific form of R(x) depends on the employed image priors.

One common problem of image denoising methods is that the image fine scale details such as texture structures will be over-smoothed [12]. Unfortunately, suppressing noise and preserving textures are difficult to achieve simultaneously, and this has been one of the most challenging problems in natural image denoising. Unlike large scale edges, the fine scale textures are much more complex and are hard to characterize by using a sparse model. To overcome this challenging problem, a gradient histogram method was developed, where these fine scale texture can be preserved and also enhanced. Texture regions in an image are homogeneous and are composed of similar local patterns, which can be characterized by using local descriptors or textons. Cognitive studies have revealed that the first-order statistics, e.g., histograms, are the most significant descriptors for texture discrimination. Meanwhile, image gradients are crucial to the perception and analysis of natural images. All these motivated to use the histogram of image gradient [15] to design new image denoising models.

The gradient histogram of image is computed with the deconvolution[10], [17] of noisy image and noise. This gradient histogram is taken as reference while restoration and fine textures that are present in image are preserved.

Suppose that we have an estimation of the gradient histogram [11] of x, denote by h_r. In order to make the gradient histogram of denoised image $\hat{x}$ nearly the same as the reference histogram h_r, then the GHP model for image denoising is as follows

$$\hat{x} = \arg\min_{x,F}\left\{\frac{1}{2\sigma^2}\|y - x\|^2 + \lambda\sum_i \|\alpha_i - \beta_i\|_1 + \mu\|F(\nabla x) - \nabla x\|^2\right\}, \qquad s.t.\, x = Do\alpha\,, h_F = h_r$$

NCSR method is used to construct the PCA sub dictionary adaptively.Based on the current estimation of image x, we cluster its patches into K clusters, and for each cluster, a PCA [16] dictionary is learned. Then for each given patch, we first check which cluster it belongs to, and then use the PCA dictionary of this cluster as D. the

GHP model is solved by using variable splitting (VS) method [13], which has been widely used in image restoration and also achieved great success.

By introducing a variable g = F (∇x), we adopt an alternating minimization strategy to update x and g alternatively. Given g = F (∇x), we update x (i.e.,) by solving the following sub-problem:

$$\hat{x} = \arg\min_{x,F}\left\{\frac{1}{2\sigma^2}\|y-x\|^2 + \lambda\sum_i \|\alpha_i - \beta_i\|_1 + \mu\|g-\nabla x\|^2\right\}, \qquad s.t.\, x = D o \alpha$$

To get the solution to the sub-problem of above equation, a gradient descent method is used to update x:

$$x^{(k+1/2)} = x^{(k)} + \delta\left(\left(\frac{1}{2\sigma^2}\left(y - x^{(k)}\right)\right) + \mu\nabla^T\left(g - \nabla x^{(k)}\right)\right)$$

$$x^{(k+1)} = D o \alpha^{(k+1)}$$

Finally the updated estimate is the denoising image.

2.3 Performance measures

Peak signal to noise ratio (PSNR)

The simplest and most widely used full-reference quality metric is the mean squared error (MSE), computed by averaging the squared intensity differences of distorted and reference image pixels, along with the related quantity of peak signal-to-noise ratio (PSNR). The Peak Signal to Noise Ratio (PSNR) is the ratio between maximum possible power and corrupting noise that affect representation of image. PSNR is usually expressed as decibel scale. The PSNR is commonly used as measure of quality reconstruction of image.

High value of PSNR indicates the high quality of image. It is most easily defined via the mean squared error (MSE) which for two m×n monochrome images I and K where one of the images is considered a noisy approximation of the other is defined as:

$$PSNR = 10\,.\log_{10}\left(\frac{MAX_I^2}{MSE}\right)$$

Here, MAX_I is the maximum possible pixel value of the image.When the pixels are represented using 8 bits per sample, this is 255.

Structural similarities (SSIM)

Natural image signals are highly structured. The motivation of SSIM [18] approach is to find a more direct way to compare the structures of the reference and the distorted signals. We define the structural information in an image as those attributes that represent the structure of objects in the scene, independent of the average luminance and contrast.

Suppose x and y are two nonnegative image signals, which have been aligned with each other (e.g., spatial patches extracted from each image). If we consider one of the signals to have perfect quality, then the similarity measure can serve as a quantitative measurement of the quality of the second signal. The system separates the task of similarity measurement into three comparisons: luminance, contrast and structure.

$$SSIM(x,y) = \frac{(2\mu_x\mu_y + C_1)(2\sigma_{xy} + C_2)}{(\mu_x^2+\mu_y^2+C_1)+(\sigma_x^2+\sigma_y^2+C_2)}$$

The above formula is used to calculate the structural similarity between two images. The maximum value of 1 says that two images are structurally equal.

Section III

Image denoising has become one of important step of image processing. Various image denoising methods are developed for removing noise from image. At time of development these gave better results when compared to their earlier methods but as research progresses various advanced methods are developed for giving better quality of denoised image. The below table I compares the merits and demerits of various image denoising methods.

Table I:Comparative table for various methods:

Methods	Advantages	Disadvantages
Total variation method	Simple and basic one	Very less SNR value
Sparsity method	Improved PSNR value and quality of denoised image	This method cannot handle similar patches of image
Non-local means method	Very good at handling similar patches and improves PSNR further	This method cannot be used if image does not have similar patches
Non-local Sparsity based method	Both similar and non-similar patches can be handled	Achieved PSNR is not satisfactory
Centralized non-local sparse method	Measurement parameters like PSNR and SSIM results are good when compared to previous methods	Smoothens the fine details of restored image
Gradient histogram method	Preserves and also enhances the fine details of restored image	PSNR can be further improved

3.1 Experimental results:

To verify the performance of Non-locally centralized sparse representation for image denoising and Gradient histogram estimation and preservation for texture enhanced image denoising, these two methods are tested with 5 natural images with various texture structures, whose scenes are shown below. All the images are gray scale images with gray level ranging from 0 to 255. For performance verification, the AWGN standard deviation is used for three different values of 20, 30 and 40.

Fig 3.1.1 Test images labeled from 1 to 5

Table III: PSNR (dB) and SSIM comparative results of NCSR and SGHP methods

Variance	σ =20		σ =30		σ =40	
Image	**NCSR**	**SGHP**	**NCSR**	**SGHP**	**NCSR**	**SGHP**
1	30.65	30.71	28.97	28.99	27.83	27.93
	0.8008	0.806	0.742	0.747	0.694	0.706
2	30.16	30.20	28.41	28.43	27.24	27.33
	0.8338	0.840	0.771	0.778	0.718	0.729
3	30.98	31	29.29	29.24	28.27	28.29
	0.813	0.815	0.754	0.756	0.709	0.716
4	28.48	28.49	26.29	26.28	24.90	24.98
	0.882	0.883	0.820	0.818	0.761	0.772
5	30.60	30.50	28.57	28.54	27.24	27.29
	0.84039	0.8393	0.782	0.7809	0.734	0.7414

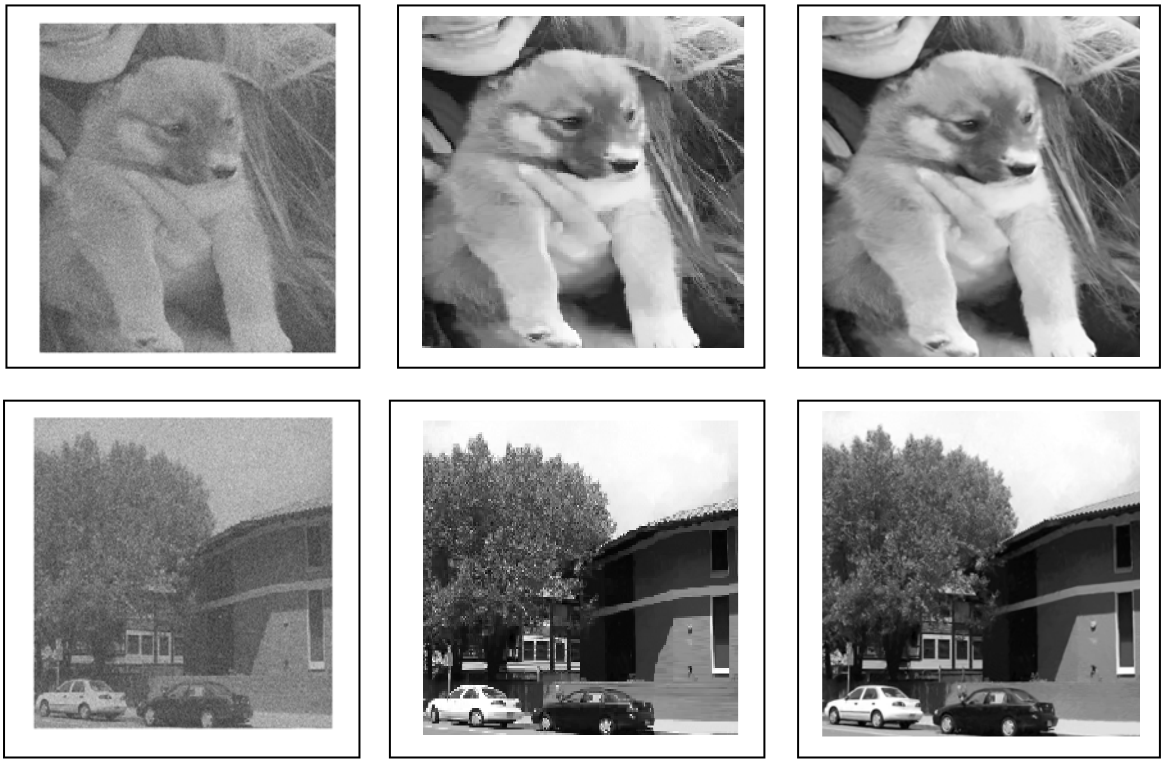


Fig 3.1.2 Noisy and Denoised images of puppy and cars for NCSR and SGHP methods

The PSNR and SSIM results of gradient histogram (SGHP) method are better when compared to centralized nonlocal sparse (NCSR) method.

3.2 Conclusion:

Various Sparsity based methods are developed for image denoising and also achieved great success. In spite of great success of these methods, research is going on for various other methods for image denoising as these earlier methods are smoothening the fine scales that are present in image. By using Gradient histogram method the fine scales that are present in image can be preserved to some extent. Further research work is going on to simplify the methods and to give better results. At present gradient histogram method is applicable for AWGN only, if any other types are present in image other than AWGN, those images with multiplicative [7] or signal dependent noise has to be transformed to an image with AWGNand then apply SGHP.So, it would be interesting to work in this direction.

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