EQUALITY IN BETWEEN ITERATIVE SOFT DILATION AND ITERATIVE SOFT CLOSE IN MULTI SCALE ENVIRONMENT

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ABSTRACT:In this paper, equality is established and discussed in between soft dilation and soft close in multi scale environment as well as in iterative environment. Soft erosion and soft dilation will exist for various thresholds. So soft open and soft close also exist for various thresholds. If definition for soft erosion and soft dilation are studied (5),then some type of equalities are viewed among soft morphological operations. So equality may be established in between softerosion and soft dilation in multi scale environment (47).open and close are composite operations.So soft open and soft close are also composite operations which will exist at various thresholds. Equality may be viewed among all soft morphological operations.

As part of that ,in this paper equality may be established in between soft dilation and soft close in multi scale as well as iterative environment.A very important point is that equality does not exist in mathematical morphology but will exist in soft mathematical morphology.

KEY WORDS: Mathematical morphology, Mathematical soft morphology, Soft morphology, Erosion, Dilation, Soft erosion, Soft dilation, Primitive morphological operation,equality,threshold , Iterative morphology, Multi scale morphology, soft open, soft close.

1. INTRODUCTION TO IMAGE PROCESSING:

IMAGE: If we observe carefully, the human beings have the desire of recording incidents, through images. Their view may be for the purpose of future generation. Images also, played the role of symbols of languages, for communication purpose.

The early cavemen documented some of the incidents through images in the caves. They documented some of the incidents of their routine life, on stones, by using primitive tools. Important incidents such as battles, routine incidents such as food habits were recorded by them, on stones. These provide record, which is historically very important, of early human civilization. The images drawn by primitive tools by Egyptians, Indians, have provided a lot of valuable information, for historians, about civilizations.

After this, paints or inks were invented. The human beings started to record scenes, incidents through these paints and inks. These people, having these capabilities may be called as artists. These artists used to accompany soldiers in battle, to record historic events. These artists used to paint religious concepts, such that, they are understood to a common man. They used to paint images/pictures of incidents of kingdoms. These images carry a lot of history to the next generations. This was started from middle ages. This discussion shows the importance of images or pictures in routine life of human race. So, the people lived in caves, people lived in Middle Ages have understood the importance of pictures (images). The desire on drawing pictures, maintain in them as treasure, and handing over to future generations is increased day by day.

Letter on J. B. Porta, an Italian Philosopher, during the II half of 18^{th} century, by mean of an accidental discovery, was able to assemble a camera like equipment by mirrors and lens, which is the first step towards the modern day photography. At the same time a France scientist observed silver chloride characteristics with respect to light. After two centuries Alexander Charles extended above concept, and produced simple photo graphs.

After one century, at around 1835 Henry Fox Talbot extended above concepts, using silver nitrate, extended the design of camera, and modern photography was born from this experiment, which is presented in royal society.

This technology is used to record incidents of U.S. civil war, or, to record incidents of wealthy people, but not reached to a common man, due to complex chemical process, for the development of photographs till “KODAK” has entered in 1884. Later on, due to research works and presentation in Royal Institute, on color systems, RGB, by James Clerk Maxwell and James D. Forbes in May 1861, a new generation in images, (Photography) was started. Due to their ground work only, now we are able to enjoy color images. Later on research is done on motion pictures by Thomas A. Edison & William Kennedy Laurie Dickson, which is foundation for modern movie technology. Actually the first step for images processing was laid during Second World War. During this period, identification of enemy targets is done using aerial photographs. But, the photographs were having inferior in quality due to poor lighting or improper weather conditions.

Technical experts, who are trained specially, are used to improve quality of image. They are specially trained in object recognition, they used to identity targets, manually. So, it is first step in image processing. After invention of digital computer, digital image processing came into existence. NASA, in early 1960's, got images from Space Crafts, Ranger 7, of the Lunar Surface, in thousands. These images were processed to minimize distortions. This is initial digital I.P. work, using a computer. This work was done in NASA's JET propulsion laboratory (JPL), in California.

This initial digital images processing work was very satisfactory. So, NASA continued it's funding, resulting in the development of digital image processing area. Because, this digital I.P. is very satisfactory in providing results, NASA extended dip to its other programs, satellite data processing. NASA launched a series of satellites–LAND SAT, SEASAT, TIROS, GEOS, NIMBUS. They used to provide multispectral images of the earth's surface. These satellites provide detailed images of surface of earth & weather information on a daily basis. This is about the beginning of DIP. At the beginning, DIP is started and applied in NASA only. Later on, these techniques are found applications in:

Medicine	Aerospace and defense
Crime and finger print analysis	Multimedia and Movie industry
Remote sensing	Manufacturing and so many other areas.

The reduction in Hardware cost, mass production of chips, reduction in memory cost, reduction in size of computers , boosted the development of Digital Image Processing area.

So, researches in general have been showing interest and developed algorithms for image smoothening, edge enhancement, image compression, image segmentation, 2D to 3D conversion etc., Now a day, it is having applications from entertainment area to medical area.

2. INTRODUCTION TO MATHEMATICAL MORPHOLOGY

At the same time mathematical morphology emerged and developed separately, with some other interests and motivations. The purpose of this area is different. But later on, it is identified that the mathematical morphology is having very important applications in image processing. So, mathematical morphology is considered now, a very important branch of image processing.

Actually J. SERRA (1) and MATHERON (2) are founders of mathematical morphology. They have explained all the fundamentals of mathematical morphology in their books.

Actually the primitive operations are EROSION & DILATION. The composite operations are open and close. All these are explained in chapters 1 and 2. There are some more composite operations, like thinning, skeletenization etc. But the work is limited to erosion, dilation, open, close.

These four operations are discussed thoroughly, with properties and proofs and extensions to gray scale in 3. Mr. H.J.A.M. HEIJMANS has given a detailed discussion of these operations in 4. Till now the light is thrown on the fundamentals of mathematical morphology (1,……. 4).

The morphological operations are suitable to apply on binary images only. But later these operations are extended to gray scale images also. One method of applying these operations on gray scale images is discussed by PETROS MARAGOS etc . They have (21) proposed a method to convert a gray scale image to binary image series. This method, named as threshold superposition, has opened new doors into this area. Morphological operations may be applied on these binary images, later on, these processed binary images are integrated to get, a processed gray scale image. So, the methodology, proposed by Maragos has extended morphological operations to gray scale environment also. They have discussed the necessary mathematical background, theorems, examples etc.

Actually, applications of morphological operations were extended by SERRA also. Later STERNBERG concentrated in this area. In depth study was done (the theoretical analysis) by J.A.M HEIGMANS (22),in this area. PETROS MARAGOS (23) has discussed about morphology also. PETROS MARAGOS (24) has discussed about morphology and given theoretical analysis.

IMANTS D. SVALBE discussed about closing in 25. The morphological operations can be implemented in various directions horizontal, vertical, and diagonal. As the S.E. size is increased, so many directions are obtained. These can be implemented, taking input as one dimensional array. In this way a new type of algorithms are developed by VAN HERIC and extended by PIERRE SOILLE, etc (26). In this paper authors explained these algorithms with sufficient mathematical back ground and good examples.

SUCHEN and HARALICK proposed (27) new types of morphological algorithms recursive erosion transforms, recursive dilation transforms, recursive open transform, recursive close transform, by using recursion concept, which is an extension to mathematical morphology. For the extension of morphological operations to gray level image, efficient algorithms are designed (28) by JOSIPH (YOSSI) Gil etc by min -max values, and these are extended for getting edges of an image.

ARNOLD MEIJSTER etc (29) designed new algorithms for mathematical morphological operations. A few researchers MOTAZ A. MOHAMED, ALDO MORALES etc have given (30,31) statistical analysis of morphological operations. They have studied new composite operations (30) like close – erosion, close – open etc and smoothening and detail preservation (31) also, with respect to statistical analysis. ROBERT L. STEVENSON (32) also dealed with statistical properties, with respect to morphological operations.

STEPHEN. S. WILSON (33) has given a different treatment with morphological operations, mixing with matrices. He maintained images as elements in a matrix. In another matrix, he maintained structuring elements as elements. He has processed the first matrix (which has images) by second matrix (which has structuring elements). He discussed the background theory and the corresponding mathematical analysis in detail, with a practical example, character identification. Definitely it is an unimaginable extension to mathematical morphology.

PETROS MARAGOS (34), has contributed excellent extension to mathematical morphology, by the name “DIFFERENTIAL MORPHOLOGY”, introducing applications of calculus, differential equations into mathematical morphology. He discussed distance transforms, and some other signal transforms, multi scale erosion, multi scale dilation also, with sufficient mathematical back ground, in this new environment. REIN VAN DEN BOOMGAARD etc also discussed similar work (43). They discussed about solving of differential equations, by morphological operations.

PIERRE SOILLE etc (35), introduced and discussed in their paper about new morphological operations TI erosion, TI dilation, TI open, TI close (here TI: Translation Invariant). NIDHAL BOUAYNAYA and others introduced (36,37) spatially – variant morphological operations, which are of new type, in binary as well as gray level environment. They have discussed SV erosion, SV dilation, AV open, SV close in detail with properties, theorems, examples. They have discussed some more composite operations also, like skeletenization segmentation etc in SV environment.

FRANK Y. SHIH etc (38) discussed about pipeline architectures for morphological operations in recursive environment. The morphological operations have extended to fuzzy area also. Using fuzzy techniques, fuzzy morphological operations are developed. (43,44,45). ISABELLED BLOCH (44) discussed fuzzy morphological operations in depth, and their applications in pattern analysis, in his works. LOUVERDIS, G & ANDREADIS, I extended (41) these fuzzy concepts to hardware implementation, in color environment.

KRISHNA MOORTHY SIVAKUMAR, HOJN GOUTSIAS extended these morphological operations to flat operations. They have designed flat erosion, flat dilation, flat open, flat close and flat structuring elements. They have applied these new morphological operations to analysis of textures. This paper provided good examples and in depth mathematical treatment for these new operations.

For elimination or minimization of noise in the images a lot of research is done. Normally a few statistical based techniques will be useful for this purpose. But morphological techniques also are useful for this purpose.

A few researchers concentrated in this area. DAN SCHONFELD etc (44) have done some research work in smoothing by morphological operations. Normally by image smoothening some useful information may be lost. But these researchers have developed algorithms, using morphological techniques, for image smoothening, without losing the important details of the image (i.e., with detail preservation). So, morphological techniques are proved to be capable for DETAIL PRESERVATION also, which is a very important IMAGE PROCESSING CHARACTERISTIC.

RONALD JONES & IMANTS SVALBE proposed (45) another method of implementation of morphological operators (Erosion, Dilation, Open, Close) and shown that the salt and pepper noise will be eliminated; (by means of their methodology). J. ALISON NOBLE has discussed about close – open, open – close (46) composite morphological operations in the context of, salt –pepper noise elimination, impulse noise elimination. [Of course he proposed these op’s for texture segmentation also]. JOHAN VAN HOREBEEK and others proposed another algorithm for noise treatment, using morphological open and close operations.

BOUAYNAYA, N; etc. (48) proposed another morphological algorithm using IDEM POTENCY and DUALTY property for elimination of speckle noise in radar images. [In this paper the importance of duality &idempotency properties are understood]. LEI, T; FAN, Y. Shown (49) elimination of impulse noise by a pair of morphological dual operators. They have shown that, this dual pairs provides better results for image smoothing.

In medical image processing, one object is identification of organs like kidneys, body cells, cells of blood etc. For these purposes, edge enhancement techniques and segmentation techniques are mainly useful. Morphological techniques are also very useful in this medical image processing because there are a few techniques, which will provide edges of the images and segments of the images. For example the following methodologies provide edges of IM's.

A) (Original Image – Eroded image)
B) (Dilated Image – Original image)
C) (Dilated Image – Eroded image)

SCHUPP, S etc (50) explained the role of morphological operations in medical I.P. Segmentation using morphological techniques is concentrated in this paper, and it is applied in medical image processing area, and it is explained with examples. YANK Y etc. demonstrated (51), applications of morphological techniques for the identification of cells.

BIN MANSOOR. A & others developed methodologies for diagnosis of diabetic (52) retinopathy, using morphological techniques. They have applied fuzzy morphology for this diagnosis. GAO YAN, BOLIANG WANG (53) proposed methodology for kidney identification using multi scale mathematical morphology. More than 200 test cases are studied using this algorithm. A QUINO, A; etc (54) developed an edge detection algorithm using morphological op's for identification of optic disc by processing of retina image.

3. SOFT MORPHOLOGY

In mathematical morphology, some type of the concept "All" will play major role.In Erosion, the O.P. will be "1", if all elements of the sub image are equal to 1, otherwise, the output will be "0". In dilation, the O.P. will be "0", if all elements of the sub image are equal to "0". Otherwise the output will be "1". This "All" concept, will cause some type of inconvenience. So some type of flexibility is introduced, in the form of threshold value. So, this morphology with threshold is defined as soft morphology. So, this soft morphology is having a few advantages, which the mathematical morphology operations don't have.

So, the Soft Morphology can be considered as extension to mathematical morphology. Even though mathematical morphological operators are efficient, they suffer with a few drawbacks as specified above. In addition to above, some more comments are………. In primitive morphological operations, erosion, one or two mismatched pixels of image prevent the structuring element from fitting perfectly. It is the basic morphological operation, quantifies the way in which, the structuring element fits into the image. Erosion is an "All or nothing" transformation, implemented using bitwise "and". So, erosion will be sensitive to noise.

In primitive morphological operations, dilation, isolated pixels, even though, they are irrelevant to the image's content, significantly affect the output of the transformation. The net effect is an increased number of large spurious particles, increasing the confusion in the dilated image. So, noise will be added, which may be named as additive noise. (5).

But, many applications require more tolerance to noise than is provided by erosion and dilation. Soft morphological operators possess many of the characteristics, which are desirable, perform better in noisy environments. (5)

So, the soft morphological filters, improve the behavior of standard morphological filters, in noisy environment. The soft morphological filters are better compared to mathematical morphology in small detail preservation and impulse noise. In soft morphology, it preserves details, by adjusting its parameters (11). It can be designed in such a way that, it performs well in removal of salt – and – pepper noise as well as Gaussian noise, simultaneously. (12)

The idea of soft morphological operations is to relax, the standard morphological definition, a little, in such a way that, a degree of Robustness is achieved, While, most of the desirable properties of standard morphological operations are maintained. The soft morphology was introduced by KOSKINEN etc, and developed by researchers.

MICHAEL A. Z MODA and LOUIS. A. TAMBURINO discussed (55) morphological operations, soft morphological operations in detail. In this paper they discussed the definitions of Erosion, Dilation on the basis of methodology like counting, which is suitable to extend to soft morphological operations, by fixing threshold values. They discussed some more algorithms for implementation of soft morphological operations, properties up to some extent. PAULI KUOSMANERI etc. (56) have discussed about statistical properties of soft morphological ops. They discussed about noise reduction using soft morphological ops, with detail preservation, in this research paper. The above authors discussed in another research paper (57) about the relation in between soft morphology ops as well as stack filters.

SHIH, F.Y. etc. discussed (58) soft morphological properties are discussed up to some extent. Some of the properties are stated and idem potency is discussed up to some extent. They discussed about, soft morphology op's in gray scale, using threshold super position theorem. They discussed about implementation of soft morphology op's, using logic gates also. Any way, it discussed soft morphological operations in a few dimensions.

PU, C.C. discussed about (59) implementation of soft morphological op's in gray scale. They integrated super position property and stacking to extend soft morphology from binary scale to gray scale.

PAULI KUOSMANEN & JAAKKO ASTOLA (60) also discussed, statistical properties, of soft morphology op's, up to some extent, with connection to stack filters.

GASTERATOS, a discussed (61) a new technique, for the realization of soft morphology op's basing upon majority gate algorithm system architecture, for implementation of soft morphology op's, is also presented. MICHAEL A. ZMUDA (62) proposed an algorithm for implementation of soft morphology ops. Normally voting logic also may be used, across neighborhoods, defined by the S.E.

But, in this algorithm instead of processing all the votes, a few votes may be choosen randomly and the service of FSM also, will be taken, in implementation of this algorithm. It is faster than conventional algorithms. Accuracy: more than 90%.

ZHAL CHUNHUI (63) designed soft morphological filter, using genetic algorithm. It is in optimized and improved algorithm. PERTTIT. KOI VISTO, etc. (64) also concentrated and discussed improved algorithms for soft morphological ops, using genetic algorithms.

M. VARDA VOULIA etc. (65) designed algorithms for small detail preservation and impulse noise suppression, using soft morphological op's [soft vector morphology] in color environment and shown better results compared to algorithm designed, based on morphological operators. [Mathematical vector morphology]. A. GASTERATOS, etc. (66) discussed about structuring element decomposition, in soft morphological environment.

A. GASTERATOS etc. (67) discussed about extension of fuzzy theory into soft morphology. G. LOUVERDIS etc. (75) also discussed about fuzzy soft morphology. These filters are less sensitive to image distortions and to small variations in the shape of the objects. Fuzzy soft morphology performs better in impulse noise removal, compared to standard morphological op's. Fuzzy soft morphology extended to edge detection also.

Soft morphological filters have entered to recursive environment also. SHIH, F.Y. & PADMAJA. P (72) PERTTI KOIVISTO etc. (71) PEI, S. etc. (76) discussed about recursive soft morphology in various contexts and environments.

In a research paper (68), the authors discussed about properties up to some extent. But, elimination of noise, as well as, detail preservation are opposite characteristics, up to some extent. A strong smoothening filter may not preserve details. But, in soft morphology, a balanced solutions maybe obtained, which will preserve details as well as suppress noise, due to flexibility in the definition of soft morphology. It is discussed by KOI VISTO, P. etc. (69).

Statistical soft morphological op's are new type of op's, which possess two types of advantages. These have properties of dealing with shape for shape preservation, due to soft morphological characteristics as well as noise cleaning properties due to statistical approach in these statistical soft morphological ops. It is introduced by STRINGA, E, etc (H_{16}). Like above methodologies, recursive order – statistic soft morphological filters/ op's, [ROSSM] balance two types of parameters. One is noise reduction. The other os detail/edge preservation. (71).

PEI, S. etc. (72) discussed these techniques and showed with examples that, these filters perform better compared to other filters like morphological filters, soft morphological filters, order–statistic soft morphological filters. A way of implementing, soft morphological op's, is discussed by LIPEND WANG etc. (73), based on graphic processing unit [GPU], reduces computing time.

Statistical soft morphology is extension to soft morphology, discussed by REGAZZONI, C.S. etc. (74), having advantages, compared to , soft morphological op's, (in image smoothening such as speckle noise handling, processing remote sensing images)

ZHENG MINGJIE etc. (76) developed directional S.E.'s for speckle noise reduction on SAR images. KOI VISTO, P; etc. (77) concentrated on detail preservation while smoothening. ZHEN JI etc. (78), designed soft morphological filter for reducing periodic noise. These results are compared with other spatial domain as well as frequency domain filters techniques.

ZHEN JI etc. (79), in another research paper, discussed about periodic noise reduction, by soft morphological filters, in another way [another algorithm].MARSHALL, S etc. (80), used soft morphological filters for elimination of disturbance, caused by solar cosmic rays, in the images obtained by astronomy base. [Solar].

In the same way smoothening, detail preservation filters basing on soft morphology are discussed in 65,75,76,68, 69,70,71,74 etc. papers, introducing extensions of soft morphological filters like statistical soft morphological filters, recursive soft morphological filters etc.

So many researchers entered in to edge detection using soft morphological op's.

HUANG FENG – GANG; etc. (81) discussed the role of soft morphological ops in edge detection, in noisy environment. ZHANG YING etc. (82) discussed about edge detection. They used PSO [Particle Swarm Optimization method] to choose best edge detection method, suitable to the environment of the image. SONG XIN LUO JUN etc. (83) discussed a method, which minimizes noise, preserve details & detects edges.

XIAOXIN GUO etc, (84), discussed a new type of filter. They integrated soft morphology, lapcacian operator as well as, nature of adaptivity. They designed, adaptive soft morphological laplacian filter, for smoothening as well as edge detection. The nature of adaptivity is achieved by, employing, 4 directional structuring elements.

Empirical mode decomposition [EMD] is a new concept in the field of signal processing. The technique, extended to analyze two dimensional data is known as bi dimensional EMD. [BEMD].

XIAOFEI YAN etc (85) proposed an edge detection method, integrating BEMD and soft morphology.

WANG TAO etc. (86) discussed an important concept they have applied soft multi – scale operations for edge enhancement in noisy environment

ST RINGA, E. etc. (87) proposed algorithm, for reconstruction of image, using soft morphology and Bayesian process and applied on SAR images. HAMID, M.S. (88) designed a multidimensional soft morphological filters in gray scale environment, using genetic algorithm for optimization for restoration. DONG YAN – ZHI etc. (89) discussed segmentation in soft morphological environment. TANLIU. Etc. (90) discussed soft morphology and top – hat tr and SPRT – PMHT for identification of targets which are small in infrared environmental images.

In some applications, like character identification, noise will be generated after shape decomposition (using morphological methods). But soft mathematical morphological methods will function excellently in this environment. It is discussed by N. SANTHI & Dr. K. RAMAR (91).

4. ITERATIVE SOFT MORPHOLOGY

It can be defined as, applying a morphological operation on an image, a few number of times.

4.1 CONVENTION:

symbolically, $(X \ominus Y)$ means applying erosion by S.E. Y, on image X.$(X \ominus 2Y)$ means, applying Erosion by S.E. Y, on image X, twice. $(X \ominus 3Y)$ means, applying Erosion by S.E. Y, on image X, thrice. $(X \ominus NY)$ means, applying Erosion by S.E. Y, on image "X", "N" number of times ,in the same way.

$(X \oplus NY)$ means, applying dilation by S.E. Y, on image "X" ,N no of times.(X O NY) means, applying open by S.E. Y, on image "X", N numbers if times. [But it is idempotent operation.] $(X \bullet NY)$ means applying close by S.E. Y, on image "X", N number of times. [But it is also idempotent operation.] This iterative morphology will have applications in the design of composite morphological operations (Morphological Algorithms) skeletenization, thinning, thickening etc.

The applications may also be seen in structuring element Decomposition, segmentation, etc.

Iterative morphology may be extended to iterative soft morphological environment also. In iterative soft morphological environment, the following convention may be used.

$(E_{(1)})^2$: Soft Erosion, with threshold value = 1 applied, 2 times on the image.

$(E_{(1)})^5$: Soft Erosion, with threshold value = 1 applied, 5 times on the image.

$(E_{(x)})^y$: Soft Erosion, with threshold value "x", applied "y" times on the image.

$E_{(1),}\ E_{(2),}\ E_{(3),}$: Soft Erosion, applied with threshold values, 1,2,3 on the image.

$E_{(x),}\ E_{(y),}E_{(z),}$: Soft Erosion, applied with threshold values, x,y,z on the image.

$(D_{(1)})^3$: Soft Dilation, with threshold value "1" applied "3" times on the image.

$(D_{(2)})^4$: Soft Dilation, with threshold value = 2, applied, "4" times on the image.

$(E_{(x)})^y$: Soft Dilation, with threshold value = x, applied "y" times on the image.

$D_{(1),}\ D_{(2),}\ D_{(3):}$ Soft Dilation, applied with threshold values, 1, 2, 3 on the image.

$D_{(x),}\ D_{(y),}\ D_{(z):}$ Soft Dilation, applied with threshold values x, y, z on the image.

$(O\ (1, 2))^3$: Soft open applied thrice on the image, with thresholds 1,2

[Soft Erosion threshold value =1, Soft Dilation threshold value =2]

$(O\ (x, y))^n$: Soft open, applied 'n" times, on the image, with thresholds x, y

[Soft Erosion threshold value = x, Soft Dilation threshold value = y]

O(p, q) O(x, y): Soft Open applied twice on the image, with different thresholds.

O (p, q) O (r, s) O (x, y): Soft open, applied thrice on the image, with different thresholds.

$(C (1, 2))^{4:}$ Soft close applied four number of times on the image, with Soft dilation threshold value = 1, Soft Erosion threshold value = 2.

$[C (1, 2)]^{n}$: Soft close applied "n" number of times, on the image with thresholds 1, 2.

$(C(x, y))^{n}$: Soft close applied "n" times, on the image, with thresholds x, y.

C(p, q) C(r, s) C(t, u): Soft close applied on the image, thrice, with different thresholds.

4.2 REVIEW ON SOFT MORPHOLOGY:

Iterative morphology means, applying one morphological operator, on an image a few no of times. These morphological operators may have same S.E or different S.E's or same S.E with different dimensions. Iterative morphology is having its own importance. It is having so many applications in so many areas.

Iterative morphology appears in skeletonization process. In an algorithm for skeletonization erosion has to be applied, a few no of times. In thinning also, iterative morphology will appear. A Structuring Element has to be applied so many times, on an image; [Each time the Structuring Element, will be rotated]. Same case in thickening also. Thickening also uses iterative morphological concept.

In some situations, multi scale iterative concept will appear. In multi scale skeletonization

S.E. will be applied at various dimensions, each time upon an image, to get skeletons at various dimensions.

In the previous section, S.E. decomposition is discussed. A S.E. will be divided into series of mini S.E,'s. All these S.E.'s will be applied on the image one after the other as a series or these can be applied on the image simultaneously in parallel computing environment. Any way structuring element decomposition deal with iterative morphology. The S.E. may be decomposed into mini S.E's, with dimensions in increasing order. So, S.E decomposition can be in iterative environment and multi scale environment (B_2) also.

CHANF – CHEF CHAOUNI, M etc developed a process for convergence criteria, in iterative morphology. In iterative morphology, the system has to go towards a better solution. It is discussed in this paper (92). XIA – YONG and others proposed an algorithm in iterative morphological environment, for segmentation, using multi fractal estimation concept, which FS suitable to be applied in remote sensing data. [Of course, this algorithm can be applied, in other images (93) also]. ROBIN, F. & others, designed H.W. SYSTEM, to implement morphological filters (operations) iteratively (94). It provides a real time processing environment.

ONGWATTANA KUL. S. & others also developed implementation of iterative morphology, in H.U. point of view (99). SHIH, F.Y. & others (100) proposed an algorithm in iterative morphological environment (Applying iterative erosion) for EUCLIDEAN distance transformations. AMAYEH, G & others (97) proposed algorithm in iterative morphological environment for hand verification.

JIWEI YUAN & others, (98) discussed methodology for segmentation technique, which is suitable to apply on sequence of images of traffic scenes. YONG XIA & others (99), discussed methodology using multi fractal estimation technique for image segmentation using iterative dilation scheme, (with a series of cubic S.E's) with reduction in computational complexity. SAARINEN, K discussed (100) segmentation, in color environment, using watershed technique in iterative morphological environment.

AUPIGITER. R. also discussed (101) segmentation using iterative watershed algorithm in 3D environment, which is suitable for medical image processing. ZHANG XIAO – JING & others discussed (102) segmentation using watershed algorithm, to be applied in medical area, using iterative erosion technique.

ZHUANG, H; & others – (103) discussed methodology for smoothing (for the treatment of impulse Gaussian noises) using iterative close – open technique. SKOLNICK, M.M etc (104) discussed determination of centroids using iterative morphology. J.G.POSTAIRE & others (105) discussed the role of iterative morphology in cluster analysis. CHIVERTON, J.P. & others, discussed (106) about applications. Of iterative morphology in medical image processing, regarding neurological analysis which is very important. JIEKANG & others (131) used iterative morphology, in CB morphological environment, on SAR images, for image Smoothing.

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5. MULTI SCALE SOFT MORPHOLOGY

5.1 DISCUSSION ON MULTI SCALE SOFT MORPHOLOGY

In the process of understanding the objective world, the appearance of an object does not depend only on the object itself, but also on the scale that the observer used. It seems that appearance under a specific scale does not give sufficient information about the essence of the percept, we want to understand. If we use a different scale, to examine this percept, it will usually have a different appearance. So, this series of images and its changing pattern over scales reflect the nature of the percept.

The S.E. dimension can be anything. It depends upon situation, requirement, and context etc. It can be $1/1, 2/2, 3/3, 4/4, 5/5, 6/6, 7/7, \ldots\ldots\ldots$

In some situations, particularly square grid is chosen, it can be $3/3, 5/5, 7/7, 9/9, 11/11, 13/13, \ldots\ldots\ldots$

The S.E.'s, having series, and in increasing size [like mentioned above] is called multi scale S.E.'s and the morphological approach (operations) dealing with multi scale S.E.'s is called multi scale morphology. As the size of the S.E. is more, its impact upon image will be more. For example, amount of expansion by applying dilation operation is more on an image, if we apply $5/5$ S.E., compared to amount of expansion of image, by dilating by $3/3$ S.E.

5.2 REVIEW ONMULTI SCALE SOFT MORPHOLOGY

Till now, some amount of research is done in this area, and it is applied in so many areas. In mathematical morphology also, a new area multi scale mathematical morphology is developed, and applied in so many areas like smoothening, edge enhancement, analysis of radar imagery, remote sensing, medical image processing etc.

PETROS MARAGOS entered into multi scale morphology, in addition to other areas. He explained about changes of shapes, as the scale is changed. He explained the applications of MSMM, and back ground mathematics. He explained about application of MSMM in skeletenization also. He extended these concepts to gray scale, also (107). MING – HUA CHEN & PING – GAN YAN explained (108) Erosion, Dilation, Open, Close in multi scale environment, with diagrams (results), mathematical analysis, as well as symbolic conventions.

PAUL. T. JACKWAY etc. (139) provided one type of analysis in MSMM. They discussed how to relate the results of one scale with the results at different scale. They have provided this analysis with good examples, using Erosion/Dilation morphological operations. This paper discussed the B.G. theory, in one angle, relating to MSMM.

KUN WANG etc. proposed an algorithm, for edge detection in the presence of Gaussian noise & salt – pepper noise in multi scale morphological environment. The experimental results are better than that of conventional algorithms (109). The same authors KUNWANG etc. proposed another algorithm for edge detection (110) which will function better in Gaussian, salt - paper noise environment, in MS morphological approach.

KIM WANG and others discussed an edge detection algorithm, in multi scale environment, which is suitable to apply on brain MRI, in noisy environment. (111).

ZENG PINGPING etc. proposed another algorithm, for edge enhancement (112) in multi scale morphological approach, using order morphology also, which is suitable to apply in noisy environment also. ZHEANHUA LI; & others (113) discussed another technique for edge enhancement, in MS morphological environment.

WENJUAN ZHANG & others, proposed another methodology for edge enhancement by combining MSMM and WAVELET transform. In this methodology, they have separated low and high frequency components, by WAVELET transform. They have applied MM techniques, MSMM techniques on low and high frequency components. These O.P.'s are integrated. It is a better method compared to, MM based or WAVELET transforms based edge detection methods, in noise environment. (114).

XOAKAI – JIAN; etc. (115) proposed another edge detection algorithm, in MS MM environment, using WAVELET transform also. CHAO LI; etc. proposed an adaptive algorithm for edge detection using multi – structure and MS MM environment. (116). PANCHAO WU & others proposed another algorithm, for edge detection in noisy environment using MS MM & WAVELET transforms.

Runway detector plays a very important role in synthetic vision system, which is helpful for pilots. But the infrared image constructed in this situation which will help pilot, will have heavy noise and bad contrast. Suitable to this situation, a multi scale morphological edge detector is proposed, which will help the pilot, for his, environmental awareness. So, MSMM is having, its applications, in aviation areas also. YANG SHANMIN and others discussed above MSMM concept. (118).

GAO LI etc (119) proposed an adaptive algorithm for edge detection of a color image (In HIS space) in MSMM environment. CHEN JIN LONG, etc. proposed another methodology for edge detection in multi structure and multi scale mathematical morphology environment (120).

HAI LONG HUANG etc. proposed an algorithm for suppression of noise and preserve edges using multi share and multi scale mathematical morphology environment (120). HAI LONG HUANG etc. proposed an algorithm for suppression of noise and preserve edges using multi share and multi scale structure elements using different directions and sizes of S.E.'s. (121)

ZHANG ANU & others have proposed another algorithm (122) for identification of weak edges in Oct images using MSMM environment, XINGHUI ZAANG etc. (123) proposed another algorithm for edge detection in color image environment, using MSMM. QING LIU etc. proposed another edge detection algorithm (124) in MSMM environment. Here, as the first step, noise is eliminated by applying open close filter in multi – scale environment. Later, edge detection algorithm is applied in MSMM environment. So, image smoothening and edge detection is done in MSMM environment. So, image smoothening and edge detection is done in MSMM environment by this algorithm, is very efficient. This algorithm is very efficient in noise elimination and nation and complex border detection. WANG TAO & WEINA (125) proposed another edge detection algorithm in MSMM environment.

Another algorithm for smoothening is discussed using MSMM. (126). XU, YANLEI; ZHAO, JIYIN discussed (127) another algorithm, for edge detection, using MSMM, in noisy environment. DANTING YUHUA CHAI, ZHAO, etc. proposed (128) an algorithm for smoothening using MSMM. XIANGZHI BAI etc. (129) proposed a new type of algorithm, for image enhancement using multi scale top-hat transform.

JIAN–HUI TAN etc. proposed (130) a new type of process using MSMM for smoothening of infrared imagery. It will have complexity due to noise. Using this methodology, they protected details also. They have taken the help of NN (Neural Net works) also. So, in this paper, MSMM & NN are integrated. JIE KANG etc. used (131) CB (Contour Bougie) morphology in MSMM environment for speckle noise treatment, in SAR (Synthetic Aperture Radar) images, for maintaining detail preservation also.

These MSMM techniques are extended to segmentation also. DEBAY LE, J. etc (132) extended MSMM for segmentation using adaptive technique and MARC DROSKE etc. also (133) used MSMM for segmentation. (134) H UANG, R. etc. discussed extension of MSMM to 3D. They discussed and designed algorithm for volume segmentation. For this purpose, they have designed spherical S. E.'s at various sizes. LETITIA, S; etc. applied MSMM for road segmentation from satellite aerial images (135).

JIANN–JONE CHEN etc. extended the MSMM to 3D segmentation, using dual (MS morphological) concepts (136).

SHU LI; etc. (137) designed water sheds segmentation algorithm, using MSMM, and applied to cell image segmentation, and got quality results. XU YING SHA; etc (138) proposed another water shed algorithm for segmentation of remote sensing images, in MSMM environment. It shows good results, by avoiding, over segmentation.

PAUL. T. JACKWAY etc. (139) provide another type of analysis in MSMM. Naturally a few questions arise in MSMM, like how to relate the results of one scale with the results of other scale. This type of analysis is provided in this paper by Erosion/Dilation operations with good examples.

J. ANDREW BANGHAM, etc. (140) discussed about decomposition, in MSMM environment using the sieve decomposition theorem / method, with good B.G. FU LIU etc. (141) discussed the methodology for identifying obstacles in lunar, using water shed method, based on MSMM. Here, for this purpose, open close operators is used, in multi scale environment. It gives better results, compared to traditional watershed method. TIE XIANG WEN etc. (142) proposed an algorithm to choose the suitable scale in multi – scale morphological top – hat transformation [this transformation is used in pattern recognition].

SHUWEI LI etc. (143) proposed method, to generate DTM and to maintain the terrain details, based on MSMM [here DTM means, Digital Terrain Model].

MSMM is having, application in medical area also. (144) DA WEI QI etc. shown an application in medical I.P. for edge detection in noisy environment, which gives better results, compared to traditional pictures. FEI ZHANG etc., given another algorithm (145), suitable for ECG analysis, in impulse noise environment using MSMM. DAWEI QI (146) proposed another algorithm, for medical analysis environment. JI – LE HU; etc.

(147) proposed another algorithm, in ECG analysis, which provided suitable and good decisions, at critical points. It is a decision making algorithm regarding heart using MSMM.

ZA BI HI, S.M etc. (148) discussed application of MSMM for retinal vessel segmentation. (149). DAWEI QI etc (150), HAI YAN GU; etc (155), WEIPING HOU etc (152) discussed the applications of MSMM in wood analysis they have done wood decay estimations, defect identification of wood, etc. RUJIANG HAO etc. (152) used MSMM open operation for identification of defects of the rolling beatings. YING ZHANG etc. (153) used MSMM to do analysis of results of turbine rotor experiment. In noise environment also, it provides good results [strong edges].

Before going to the main concept better to refer papers of author 150,149,148.

6. EQUALITY IN BETWEEN ITERATIVE SOFT DILATION AND ITERATIVE SOFT CLOSE IN MULTI SCALE ENVIRONMENT

6.1 In this paper equality in between soft erosion and soft dilation has to be applied in various contexts. It is discussed in author's paper.(152)..In that paper equality is discussed thoroughly from basics,

So in 3/3 environment

$E(1)=D(9)$ $E(2)=D(8)$ $E(3)=D(7)$ $E(4)=D(6)$ $E(5)=D(5)$
$E(6)=D(4)$ $E(7)=D(3)$ $E(8)=D(2)$ $E(9)=D(1)$

In general, $E(m)=D(10-m)$ where m will run from 1 to 9, the threshold value.In the same way.

$D(1)=E(9)$ $D(2)=E(8)$ $D(3)=E(7)$ $D(4)=E(6)$ $D(5)=E(5)$
$D(6)=E(4)$ $D(7)=E(3)$ $D(8)=E(2)$ $D(9)=E(1)$

In general, $\mathbf{D(m)=E(10-m)}$ where m will run from 1 to 9, the threshold value.

In the same way in 5/5 environment

$E(1)=D(25)$ $E(2)=D(24)$ $E(3)=D(23)$ $E(4)=D(22)$
$E(5)=D(21)$ $E(6)=D(20)$ $E(7)=D(19)$ $E(8)=D(18)$
$E(9)=D(17)$ $E(10)=D(16)$ $E(11)=D(15)$ $E(12)=D(14)$
$E(13)=D(13)$ $E(14)=D(12)$ $E(15)=D(11)$
$E(16)=D(10)$
$E(17)=D(9)$ $E(18)=D(8)$ $E(19)=D(7)$ $E(20)=D(6)$
$E(21)=D(5)$ $E(22)=D(4)$ $E(23)=D(3)$ $E(24)=D(2)$
$E(25)=D(1)$

In general, $E(m)=D(26-m)$ where m will run from 1 to 25, the threshold value.

In the same way

$D(1)=E(25)$ $D(10)=E(16)$ $D(18)=E(8)$
$D(2)=E(24)$ $D(11)=E(15)$ $D(19)=E(7)$
$D(3)=E(23)$ $D(12)=E(14)$ $D(20)=E(6)$
$D(4)=E(22)$ $D(13)=E(13)$ $D(21)=E(5)$
$D(5)=E(21)$ $D(14)=E(12)$ $D(22)=E(4)$
$D(6)=E(20)$ $D(15)=E(11)$ $D(23)=E(3)$
$D(7)=E(19)$ $D(16)=E(10)$ $D(24)=E(2)$
$D(8)=E(18)$ $D(17)=E(9)$ $D(25)=E(1)$
$D(9)=E(17)$

In general, $D(m)=E(26-m)$ where m will run from 1 to 25, the threshold value.

In the same way in 7/7 environment

$E(1)=D(49)$
$E(2)=D(48)$
$E(3)=D(47)$
$E(4)=D(46)$
$E(5)=D(45)$
$E(6)=D(44)$
$E(7)=D(43)$
$E(8)=D(42)$
$E(9)=D(41)$
$E(10)=D(40)$
$E(11)=D(39)$
$E(12)=D(38)$
$E(13)=D(37)$
$E(14)=D(36)$
$E(15)=D(35)$
$E(16)=D(34)$
$E(17)=D(33)$
$E(18)=D(32)$
$E(19)=D(31)$
$E(20)=D(30)$
$E(21)=D(29)$
$E(22)=D(28)$
$E(23)=D(27)$
$E(24)=D(26)$
$E(25)=D(25)$
$E(26)=D(24)$
$E(27)=D(23)$
$E(28)=D(22)$
$E(29)=D(21)$
$E(30)=D(20)$
$E(31)=D(19)$
$E(32)=D(18)$
$E(33)=D(17)$
$E(34)=D(16)$
$E(35)=D(15)$
$E(36)=D(14)$
$E(37)=D(13)$
$E(38)=D(12)$
$E(39)=D(11)$
$E(40)=D(10)$
$E(41)=D(9)$
$E(42)=D(8)$
$E(43)=D(7)$
$E(44)=D(6)$
$E(45)=D(5)$
$E(46)=D(4)$
$E(47)=D(3)$
$E(48)=D(2)$
$E(49)=D(1)$

In general, $\underline{E(m)=D(50-m)}$ where m = 1 to 49.

In the same way we can have the equalities like $D(1)=E(49)$

$D(2)=E(48)$
$D(3)=E(47)$
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……………
……………
$D(48)=E(2)$
$D(49)=E(1)$

So $\underline{D(m)=E(50-m)}$ where m = 1 to 49.

The same type of discussion may be extended to 9/9,11/11,13/13,15/15 ………

InGeneral

For structuring element size: w/w

- $E(m)=D(w^2+1-m)$

By the same logic $D(m)=E(w^2+1-m)$

6.2 In this paper the applications of equalities in between soft open and soft close is also applied which is discussed in author's paper(151).

In 3/3 environment

$$O(1,1)=E(1)D(1)=D(9)E(9)=C(9,9)$$
$$O(1,2)=E(1)D(2)=D(9)E(8)=C(9,8)$$
$$O(1,3)=E(1)D(3)=D(9)E(7)=C(9,7)$$
$$O(1,4)=E(1)D(4)=D(9)E(6)=C(9,6)$$

$$O(1,5) = E(1)D(5) = D(9)E(5) = C(9,5)$$
$$O(1,6) = E(1)D(6) = D(9)E(4) = C(9,4)$$
$$O(1,7) = E(1)D(7) = D(9)E(3) = C(9,3)$$
$$O(1,8) = E(1)D(8) = D(9)E(2) = C(9,2)$$
$$O(1,9) = E(1)D(9) = D(9)E(1) = C(9,1)$$

$$O(2,1) = E(2)D(1) = D(8)E(9) = C(8,9)$$
$$O(2,2) = E(2)D(2) = D(8)E(8) = C(8,8)$$
$$O(2,3) = E(2)D(3) = D(8)E(7) = C(8,7)$$
$$O(2,4) = E(2)D(4) = D(8)E(6) = C(8,6)$$
$$O(2,5) = E(2)D(5) = D(8)E(5) = C(8,5)$$
$$O(2,6) = E(2)D(6) = D(8)E(4) = C(8,4)$$
$$O(2,7) = E(2)D(7) = D(8)E(3) = C(8,3)$$
$$O(2,8) = E(2)D(8) = D(8)E(2) = C(8,2)$$
$$O(2,9) = E(2)D(9) = D(8)E(1) = C(8,1)$$

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$$O(9,1) = E(9)D(1) = D(1)E(9) = C(1,9)$$
$$O(9,2) = E(9)D(2) = D(1)E(8) = C(1,8)$$
$$O(9,3) = E(9)D(3) = D(1)E(7) = C(1,7)$$
$$O(9,4) = E(9)D(4) = D(1)E(6) = C(1,6)$$
$$O(9,5) = E(9)D(5) = D(1)E(5) = C(1,5)$$
$$O(9,6) = E(9)D(6) = D(1)E(4) = C(1,4)$$
$$O(9,7) = E(9)D(7) = D(1)E(3) = C(1,3)$$
$$O(9,8) = E(9)D(8) = D(1)E(2) = C(1,2)$$
$$O(9,9) = E(9)D(9) = D(1)E(1) = C(1,1)$$

In general $\underline{O(m,n) = C(10-m, 10-n)}$

In 5/5 environment

$$O(1,1) = E(1)D(1) = D(25)E(25) = C(25,25)$$
$$O(1,2) = E(1)D(2) = D(25)E(24) = C(25,24)$$
$$O(1,3) = E(1)D(3) = D(25)E(23) = C(25,23)$$
$$O(1,4) = E(1)D(4) = D(25)E(22) = C(25,22)$$
$$O(1,5) = E(1)D(5) = D(25)E(21) = C(25,21)$$
$$O(1,6) = E(1)D(6) = D(25)E(20) = C(25,20)$$
$$O(1,7) = E(1)D(7) = D(25)E(19) = C(25,19)$$
$$O(1,8) = E(1)D(8) = D(25)E(18) = C(25,18)$$
$$O(1,9) = E(1)D(9) = D(25)E(17) = C(25,17)$$
$$O(1,10) = E(1)D(10) = D(25)E(16) = C(25,16)$$
$$O(1,11) = E(1)D(11) = D(25)E(15) = C(25,15)$$
$$O(1,12) = E(1)D(12) = D(25)E(14) = C(25,14)$$
$$O(1,13) = E(1)D(13) = D(25)E(13) = C(25,13)$$
$$O(1,14) = E(1)D(14) = D(25)E(12) = C(25,12)$$
$$O(1,15) = E(1)D(15) = D(25)E(11) = C(25,11)$$
$$O(1,16) = E(1)D(16) = D(25)E(10) = C(25,10)$$
$$O(1,17) = E(1)D(17) = D(25)E(9) = C(25,9)$$

$$O(1,18) = E(1)D(18) = D(25)E(8) = C(25,8)$$
$$O(1,19) = E(1)D(19) = D(25)E(7) = C(25,7)$$
$$O(1,20) = E(1)D(20) = D(25)E(6) = C(25,6)$$
$$O(1,21) = E(1)D(21) = D(25)E(5) = C(25,5)$$
$$O(1,22) = E(1)D(22) = D(25)E(4) = C(25,4)$$
$$O(1,23) = E(1)D(23) = D(25)E(3) = C(25,3)$$
$$O(1,24) = E(1)D(24) = D(25)E(2) = C(25,2)$$
$$O(1,25) = E(1)D(25) = D(25)E(1) = C(25,1)$$

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$$O(25,1) = E(25)D(1) = D(1)E(25) = C(1,25)$$
$$O(25,2) = E(25)D(2) = D(1)E(24) = C(1,24)$$
$$O(25,3) = E(25)D(3) = D(1)E(23) = C(1,23)$$
$$O(25,4) = E(25)D(4) = D(1)E(22) = C(1,22)$$
$$O(25,5) = E(25)D(5) = D(1)E(21) = C(1,21)$$
$$O(25,6) = E(25)D(6) = D(1)E(20) = C(1,20)$$
$$O(25,7) = E(25)D(7) = D(1)E(19) = C(1,19)$$
$$O(25,8) = E(25)D(8) = D(1)E(18) = C(1,18)$$
$$O(25,9) = E(25)D(9) = D(1)E(17) = C(1,17)$$
$$O(25,10) = E(25)D(10) = D(1)E(16) = C(1,16)$$
$$O(25,11) = E(25)D(11) = D(1)E(15) = C(1,15)$$
$$O(25,12) = E(25)D(12) = D(1)E(14) = C(1,14)$$
$$O(25,13) = E(25)D(13) = D(1)E(13) = C(1,13)$$
$$O(25,14) = E(25)D(14) = D(1)E(12) = C(1,12)$$
$$O(25,15) = E(25)D(15) = D(1)E(11) = C(1,11)$$
$$O(25,16) = E(25)D(16) = D(1)E(10) = C(1,10)$$
$$O(25,17) = E(25)D(17) = D(1)E(9) = C(1,9)$$
$$O(25,18) = E(25)D(18) = D(1)E(8) = C(1,8)$$
$$O(25,19) = E(25)D(19) = D(1)E(7) = C(1,7)$$
$$O(25,20) = E(25)D(20) = D(1)E(6) = C(1,6)$$
$$O(25,21) = E(25)D(21) = D(1)E(5) = C(1,5)$$
$$O(25,22) = E(25)D(22) = D(1)E(4) = C(1,4)$$
$$O(25,23) = E(25)D(23) = D(1)E(3) = C(1,3)$$
$$O(25,24) = E(25)D(24) = D(1)E(2) = C(1,2)$$
$$O(25,25) = E(25)D(25) = D(1)E(1) = C(1,1)$$

In general, $\underline{O(m,n) = C(26-m, 26-n)}$

In 7/7 environment

$$O(1,1) = E(1)D(1) = D(49)E(49) = C(49,49)$$
$$O(1,2) = E(1)D(2) = D(49)E(48) = C(49,48)$$
$$O(1,3) = E(1)D(3) = D(49)E(47) = C(49,47)$$
$$O(1,4) = E(1)D(4) = D(49)E(46) = C(49,46)$$
$$O(1,5) = E(1)D(5) = D(49)E(45) = C(49,45)$$
$$O(1,6) = E(1)D(6) = D(49)E(44) = C(49,44)$$
$$O(1,7) = E(1)D(7) = D(49)E(43) = C(49,43)$$
$$O(1,8) = E(1)D(8) = D(49)E(42) = C(49,42)$$

$$O(1,9) = E(1)D(9) = D(49)E(41) = C(49,41)$$
$$O(1,10) = E(1)D(10) = D(49)E(40) = C(49,40)$$

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$$O(1,40) = E(1)D(40) = D(49)E(10) = C(49,10)$$
$$O(1,41) = E(1)D(41) = D(49)E(9) = C(49,9)$$
$$O(1,42) = E(1)D(42) = D(49)E(8) = C(49,8)$$
$$O(1,43) = E(1)D(43) = D(49)E(7) = C(49,7)$$
$$O(1,44) = E(1)D(44) = D(49)E(6) = C(49,6)$$
$$O(1,45) = E(1)D(45) = D(49)E(5) = C(49,5)$$
$$O(1,46) = E(1)D(46) = D(49)E(4) = C(49,4)$$
$$O(1,47) = E(1)D(47) = D(49)E(3) = C(49,3)$$
$$O(1,48) = E(1)D(48) = D(49)E(2) = C(49,2)$$
$$O(1,49) = E(1)D(49) = D(49)E(1) = C(49,1)$$

$$O(49,1) = E(49)D(1) = D(1)E(49) = C(1,49)$$
$$O(49,2) = E(49)D(2) = D(1)E(48) = C(1,48)$$
$$O(49,3) = E(49)D(3) = D(1)E(47) = C(1,47)$$
$$O(49,4) = E(49)D(4) = D(1)E(46) = C(1,46)$$
$$O(49,5) = E(49)D(5) = D(1)E(45) = C(1,45)$$
$$O(49,6) = E(49)D(6) = D(1)E(44) = C(1,44)$$
$$O(49,7) = E(49)D(7) = D(1)E(43) = C(1,43)$$
$$O(49,8) = E(49)D(8) = D(1)E(42) = C(1,42)$$
$$O(49,9) = E(49)D(9) = D(1)E(41) = C(1,41)$$
$$O(49,10) = E(49)D(10) = D(1)E(40) = C(1,40)$$

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$$O(49,40) = E(49)D(40) = D(1)E(10) = C(1,10)$$
$$O(49,41) = E(49)D(41) = D(1)E(9) = C(1,9)$$
$$O(49,42) = E(49)D(42) = D(1)E(8) = C(1,8)$$
$$O(49,43) = E(49)D(43) = D(1)E(7) = C(1,7)$$
$$O(49,44) = E(49)D(44) = D(1)E(6) = C(1,6)$$
$$O(49,45) = E(49)D(45) = D(1)E(5) = C(1,5)$$
$$O(49,46) = E(49)D(46) = D(1)E(4) = C(1,4)$$
$$O(49,47) = E(49)D(47) = D(1)E(3) = C(1,3)$$
$$O(49,48) = E(49)D(48) = D(1)E(2) = C(1,2)$$
$$O(49,49) = E(49)D(49) = D(1)E(1) = C(1,1)$$

In general, $\underline{O(m,n) = C(49-m,49-n)}$

This discussion may be extended to 9/9,11/11.13/13,15/15…….

In general

For W/W structuring element size

- $O(m,n) = C(w^2+1-m, w^2+1-n)$In the same way we get

For W/W structuring element size

- $C(m,n) = O(w^2+1-m, w^2+1-n)$

6.3 In this section the equality of soft dilation and soft close is established applying above mentioned info.

6.3.1 $^{3}\!/_{3}$ Structuring Element.

In general ,let threshold value = m

$$\left.\begin{matrix} Formula\ for \\ iterative \\ soft\ dilation \\ applied \\ 2n\ times\ on \\ an\ image \end{matrix}\right\} = \begin{pmatrix} D(m)D(m)D(m)D(m)D(m)D(m) \ldots\ldots D(m)D(m) \\ 1 \qquad 2 \qquad 3 \qquad \ldots\ldots \qquad n \end{pmatrix}$$

$$= D(m)E(10-m)D(m)E(10-m)D(m)E(10-m)D(m)E(10-m)\ldots\ldots D(m)E(10-m)$$

$$(\because D(m) = E(10-m))$$

$= C(m,10-m)C(m,10-m)C(m,10-m)C(m,10-m)\ldots\ldots C(m,10-m)$

$= (C(m,10-m))^n$

$\therefore (D(m))^{2n} = (C(m,10-m))^n$

6.3.2 $^{5}\!/_{5}$ Structuring Element.

In general, let threshold value = m

$$\left.\begin{matrix} Formula\ for \\ iterative \\ soft\ dilation \\ applied \\ 2n\ times\ on \\ an\ image \end{matrix}\right\} = \begin{pmatrix} D(m)D(m)D(m)D(m)D(m)D(m) \ldots\ldots D(m)D(m) \\ 1 \qquad 2 \qquad 3 \qquad \ldots\ldots \qquad n \end{pmatrix}$$

$$= D(m)E(26-m)D(m)E(26-m)D(m)E(26-m)D(m)E(26-m)\ldots\ldots D(m)E(26-m)$$

$$(\because D(m) = E(26-m))$$

$= C(m,26-m)C(m,26-m)C(m,26-m)C(m,26-m)\ldots\ldots C(m,26-m)$

$= (C(m,26-m))^n$

$$\therefore (D(m))^{2n} = (C(m,26-m))^n$$

6.3.3 $^{7}\!/_{7}$ Structuring Element.

In general, let threshold value = m

$$\left.\begin{matrix} Formula\ for \\ iterative \\ soft\ dilation \\ applied \\ 2n\ times\ on \\ an\ image \end{matrix}\right\} = \begin{pmatrix} D(m)D(m)D(m)D(m)D(m)D(m) \ldots\ldots D(m)D(m) \\ 1 \qquad 2 \qquad 3 \qquad \ldots\ldots \qquad n \end{pmatrix}$$

$$= D(m)E(50-m)D(m)E(50-m)D(m)E(50-m)D(m)E(50-m)\ldots\ldots D(m)E(50-m)$$

$$(\because D(m) = E(50-m))$$

$= C(m,50-m)C(m,50-m)C(m,50-m)C(m,50-m)\ldots\ldots C(m,50-m)$

$= (C(m,50-m))^n$

$$\therefore (D(m))^{2n} = (C(m,50-m))^n$$

6.3.4 $^{9}\!/_{9}$ Structuring Element.

For this ,elaborated discussion is given.

6.3.4.1 Th=1:

Threshold value fixed at 1.

$$\left.\begin{matrix} Formula\ for\ iterative\ soft\ dilation \\ applied\ twice\ on\ an\ image \end{matrix}\right\} = (D(1)D(1))$$

$= (D(1)\ E(81))$ $[\because D(1) = E(81)]$

$= C(1,81)$

$\therefore (D(1))^2 = C(1,81)$

$$\left.\begin{matrix} Formula\ for\ iterative\ soft\ dilation \\ applied\ four\ times\ on\ an\ image \end{matrix}\right\} = (D(1)D(1)D(1)D(1))$$

$= (D(1)\ E(81)D(1)\ E(81))$ $[\because D(1) = E(81)]$

$= (C(1,81)C(1,81))$

$= (C(1,81))^2$

$\therefore (D(1))^4 = (C(1,81))^2$

$$\left.\begin{matrix} Formula\ for\ iterative\ soft\ dilation \\ applied\ six\ times\ on\ an\ image \end{matrix}\right\} = (D(1)D(1)D(1)D(1)D(1)D(1))$$

$= (D(1)\ E(81)D(1)\ E(81)D(1)\ E(81))$ $[\because D(1) = E(81)]$

$= (C(1,81)C(1,81)C(1,81))$

$= (C(1,81))^3$

$\therefore (D(1))^6 = (C(1,81))^3$

$$\left.\begin{matrix} Formula\ for \\ iterative\ soft\ dilation \\ applied \\ ten\ times\ on \\ an\ image \end{matrix}\right\} = (D(1)D(1)D(1)D(1)D(1)D(1)D(1)D(1)D(1)D(1))$$

$= \big(\ D(1)E(81)\ D(1)E(81)\ D(1)E(81)\ D(1)E(81)\ D(1)\big)E(81)$ $[\because D(1) = E(81)]$

$= (C(1,81)C(1,81)C(1,81)C(1,81)C(1,81))$

$= (C(1,81))^5$

$\therefore (D(1))^{10} = (C(1,81))^5$

$$\left.\begin{matrix} Formula\ for \\ iterative \\ soft\ dilation \\ applied \\ 2n\ times\ on \\ an\ image \end{matrix}\right\} = \underset{1\qquad\quad 2\qquad\quad 3\qquad \dots\dots \qquad n}{\big(D(1)D(1)D(1)D(1)D(1)D(1)\dots\dots D(1)D(1)\big)}$$

$$= \underset{1\qquad\quad 2\qquad\quad 3\qquad \dots\dots \qquad n}{\big(D(1)E(81)D(1)E(81)D(1)\dots\dots E(81)D(1)E(81)\big)} \qquad [\because D(1) = E(81)]$$

$$= \underset{1\qquad 2\qquad 3\qquad \dots\dots \qquad n}{\big(C(1,81)C(1,81)C(1,81)\dots\dots C(1,81)\big)}$$

$= (C(1,81))^n$

$\therefore (D(1))^{2n} = (C(1,81))^n$

6.3.4.2 Th=60:

Threshold value fixed at 60.

$$\left.\begin{matrix} Formula\ for\ iterative\ soft\ dilation \\ applied\ twice\ on\ an\ image \end{matrix}\right\} = (D(60)D(60))$$

$= (D(60)E(22))$ $[\because D(60) = E(22)]$

$= C(60,22)$

$\therefore (D(60))^2 = C(60,22)$

$$\left.\begin{matrix} Formula\ for\ iterative\ soft\ dilation \\ applied\ four\ times\ on\ an\ image \end{matrix}\right\} = (D(60)D(60)D(60)D(60))$$

$= \big(\ D(60)E(22)\ D(60)E(22)\big)$ $[\because D(60) = E(22)]$

$= (C(60,22)C(60,22))$

$= (C(60,22))^2$

$\therefore (D(60))^4 = (C(60,22))^2$

$$\left.\begin{matrix} Formula\ for\ iterative\ soft\ dilation \\ applied\ six\ times\ on\ an\ image \end{matrix}\right\} = (D(60)D(60)D(60)D(60)D(60)D(60))$$

$= \big(D(60)E(22)\ D(60)E(22)\ D(60)E(22)\big) \qquad [\because D(60) = E(22)]$

$= (C(60,22)C(60,22)C(60,22))$

$= (C(60,22))^3$

$\therefore (D(60))^6 = (C(60,22))^3$

$$\left.\begin{matrix} Formula\ for \\ iterative \\ soft\ dilation \\ applied \\ ten\ times\ on \\ an\ image \end{matrix}\right\} = (D(60)D(60)D(60)D(60)D(60)D(60)D(60)D(60)D(60)D(60))$$

$= \big(D(60)E(22)\ D(60)E(22)\ D(60)E(22)\ D(60)E(22)\ D(60)E(22)\big)$

$[\because D(60) = E(22)]$

$= (C(60,22)C(60,22)C(60,22)C(60,22)C(60,22))$

$= (C(60,22))^5$

$\therefore (D(60))^{10} = (C(60,22))^5$

$$\left.\begin{matrix} Formula\ for \\ iterative \\ soft\ dilation \\ applied \\ 2n\ times\ on \\ an\ image \end{matrix}\right\} = \underset{1 \qquad\quad 2 \qquad\quad 3 \qquad\quad \dots\dots \qquad\quad n}{\big(D(60)D(60)D(60)D(60)D(60)D(60) \dots\dots D(60)D(60)\big)}$$

$$= \underset{1 \qquad\quad 2 \qquad\quad 3 \qquad\quad \dots\dots \qquad\quad n}{\big(D(60)E(22)D(60)E(22)D(60) \dots\dots E(22)D(60)E(22)\big)}$$

$[\because D(60) = E(22)]$

$$= \underset{1 \qquad 2 \qquad 3 \qquad \dots\dots \qquad n}{\big(C(60,22)C(60,22)C(60,22) \dots\dots C(60,22)\big)}$$

$= (C(60,22))^n$

$\therefore (D(60))^{2n} = (C(60,22))^n$

6.3.4.3 Th=m:

In general let threshold value = m

$$\left.\begin{matrix} Formula\ for \\ iterative \\ soft\ dilation \\ applied \\ 2n\ times\ on \\ an\ image \end{matrix}\right\} = \underset{1 \qquad\quad 2 \qquad\quad 3 \qquad\quad \dots\dots \qquad\quad n}{\big(D(m)D(m)D(m)D(m)D(m)D(m) \dots\dots D(m)D(m)\big)}$$

$= D(m)E(82-m)D(m)E(82-m)D(m)E(82-m)D(m) \dots\dots$

$E(82-m)D(m)E(82-m)$

$= C(m,82-m)C(m,82-m)C(m,82-m)C(m,82-m) \dots\dots C(m,82-m)$

$\therefore (D(m))^{2n}$=$(C(m,82-m))^n$

In the same way the formulae may be developed for 11 x 11, 13 x 13, 15 x 15, 17 x 17.... structuring elements.

6.3.5 General case: $W/_{W}$ Structuring Element.

$$\left.\begin{matrix} Formula\ for \\ iterative \\ soft\ dilation \\ applied \\ 2n\ times\ on \\ an\ image \end{matrix}\right\} = \underset{1 \qquad\quad 2 \qquad\quad 3 \qquad\quad \dots\dots \qquad\quad 2n}{\big(D(m)D(m)D(m)D(m)D(m)D(m) \dots\dots D(m)D(m)\big)}$$

$$= D(m)E(w^2+1-m)D(m)\ldots\ldots E(w^2+1-m)D(m)E(w^2+1-m)$$
$$(\because D(m) = E(w^2+1-m))$$

$$= C(m, w^2+1-m)C(m, w^2+1-m)\ldots\ldots C(m, w^2+1-m)$$
$$= (C(m, w^2+1-m))^n$$

- $\therefore (D(m))^{2n} = (C(m, w^2+1-m))^n$

7. CONCLUSION

In this paper a fundamental rule called EQUALITY is discussed in multi scale and iterative environment. It will fill up gap ,on the fundamentals of mathematical soft morphology. Till now applications are discussed in various papers by various researchers,but fundamental properties are not touched. More over iterative morphology is having broad applications. so discussion of fundamental property in this context ,will lead to development of this area. Understanding of fundamental properties of any area will lead to development and expansion of that area, which will lead to excellent applications.

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