Advantages of Blackman Window over Hamming Window Method for designing FIR Filter

Subhadeep Chakraborty

Department of Electronics and Communication Engineering
Calcutta Institute of Technology
subha.journal@gmail.com

Abstract—In signal processing, there are mainly two types of filters exist and the are Finite Impulse Response(FIR) filter and Infinite Impulse Response(IIR) filter. Finite Impulse Response(FIR) filter can be designed form Infinite Impulse Response(IIR) filter by various techniques. The widely used technique is the window technique. In this paper, the designing analysis of FIR filter are shown by Blackman window and the Rectangular window. The output of the FIR design by Blackman window and the Rectangular window are shown in this paper by simulating the code in Matlab 7. The matlab program returns with a satisfactory result with proper magnitude plotting.

Keywords- FIR filter, Blackman window, Hamming window, Realization, coefficient

I. INTRODUCTION

Filter is an essential part in Digital Signal Processing (DSP). Different types of filters are used for different purposes like lowpass filters are used to pass the low frequency band whereas the highpass filters are used to pass the high frequency band. Typically there are two types of filters and they are the Finite Impulse Response(FIR) filter and Infinite Impulse Response(IIR) filter[1][3][26][27]. In signal processing, FIR filter is such type of filter whose impulse response is of finite duration as the impulse response of FIR filter is settled to zero at infinite time. While implementation, FIR filter needs no feedback i.e. FIR filter is not a recursive filter and for this reason the construction of FIR filter is much more simpler than compared to the IIR filter[2][4][5][23][28].

For designing the FIR filter, the analog filter is first constructed by active or passive elemnents. The analog filter is then mapped suitably into digital domain accuring the required IIR filter. Then by applying proper method, generally applying Fourier series method or Window method, the FIR filter can be obtained[3][6][7][18][21][25][27]. Basically, for designing, window methods are widely used. Blackman window and the Hamming window are the two window methods among all the windows, are discussed in this paper for designing the FIR filter. The comparisons in between the designing of FIR filter based on Blackman window and Hamming window are also discussed as there are certain advantages of Blackman window over the Hamming window are evident[7][8][9][19][20][26]. A comparative discussion is presented here along with the proper simulation of FIR filter by Blackman window and the Hamming window.

II. FIR FILTER

Finite Impulse Response (FIR) filter have some properties for which the designers prefer FIR filter over the IIR filter. The impulse response of FIR filter is of finite duration but it will be of infinite duration for IIR filter. There are some types of FIR filter just same as the IIR filter like[4][9][10][22][27][28]:

1. Lowpass filter
2. Highpass filter
3. Bandpass filter
4. Bandstop filter
5. Allpass filter
6. Comb filter

The advantages of FIRfilter over IIR filter are given below[4][6][7][18][21][27]:

1. FIR filters are stable.
2. FIR filters can be easily designed as for it's linear phase.
3. FIR filters, when implemented on a finite word length digital system, are free of limit cycle oscillations.
4. There are various methods are available for designing the FIR filter.

5. FIR filter require no feedback i.e any rounding error are not compounded by summed iteration and for that it is inherently stable.

6. Impulse response is finite.

7. FIR filter obeys the rules of Kronecker delta rule which says the impulse response will be zero after a finite time and hence the Kronecker delta is defined by,

$$\delta_{ij} = \begin{cases} 1, & if \quad i = j \\ 0, & if \quad i \neq j \end{cases} \qquad ....(1.1)$$

Now eqn (1.1), is valid only for two variables i.e, i and j. If there one variable exists, the Kronecker delta becomes,

$$\delta_i = \begin{cases} 1, & if \quad i = 0 \\ 0, & if \quad i \neq 0 \end{cases} \qquad ....(1.2)$$

The same concept is applicable in signal processing in impulse response i.e,

$$\delta[n] = \begin{cases} 1, & if \quad n = 0 \\ 0, & if \quad n \neq 0 \end{cases} \qquad ....(1.3)$$

From eqn (1.3), it can be said that the impulse response will be zero after some finite time where n is finite. This concept is applicable in FIR filter resulting that the impulse response of FIR filter is finite i.e zero after some finite time of finite period[23][24][27].

III. TAP WEIGHT

The output of a discrete-time FIR system is described by the weighted sum of previous inputs and the current input[11][17][26][27]. If the input to the system be x[n] and the output be y[n], the y[n] can be described by following equation:

$$y[n] = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2] + ... + b_N x[n-N]$$

$$= \sum_{i=0}^{N} b_i x[n-i] \qquad ....(1.4)$$

where,

x[n] = input signal

y[n] = output signal

b_i = Tap weight or more generally filter coefficient

N   = Order of filter

IV. FOURIER SERIES METHOD

The frequency response $H\left(e^{j\omega}\right)$ of a system is periodic in 2π. In fourier series analysis, a periodic function can be expressed as a linear combination of complex exponential. The desired frequency response of a FIR filter can be expressed by,

$$H_d\left(e^{j\omega}\right) = \sum_{n=-\infty}^{\infty} h_d(n) e^{-j\omega n} \qquad ....(1.5)$$

where,

$h_d(n)$ = Fourier coefficient

= desired impulse response

Hence, the desired impulse response can be expressed by,

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d\left(e^{j\omega}\right) e^{j\omega n} d\omega \qquad ....(1.6)$$

The z-transform of $h_d(n)$ is given by,

$$H(z) = \sum_{n=-\infty}^{\infty} h_d(n) z^{-n} \qquad ....(1.7)$$

Now, $h_d(n)$ is very essential in determining the transfer function of FIR filter as well as the detremination of the filter coefficients which in turns determines the characteristics of the FIR filter.

V. WINDOW TECHNIQUE

Window technique involves a function called window function or apodization function which states that if some interval is choosen, it returns with finite non-zero value inside that interval and zero value outside that interval. So, if the window with choosen interval is applied on a IIR system, it will obviously return with a finite non-zero value inside that interval producing a FIR system and all other value that are outside the interval will be zero. So, We can view the finite response inside some predefined interval[1][10][16][18][19][28].

There are many window techniques available for designing the FIR filter and they are[22][23][26][28]:

1. Blackman window
2. Hamming window
3. Rectangular window
4. Triangular window or Bartlett window
5. Hanning window
6. B-Spline window
7. Parzen window
8. Welch window
9. Raised cosine window
10. Kaiser window

There are also some compund type windows available for designing the FIR filter and they are[18][20][25][27][28]:

1. Nuttall window
2. Blackman-Nuttall window
3. Blackman-Haris window
4. Flat-tor window
5. Rife-Vincent window
6. Power of Cosine window
7. Gaussian window
8. Tukey window
9. Planck-Taper window
10. Slepian window
11. Dolph-Chebyshev window
12. Poisson window
13. Bartlett-Hann window

Hence, in this paper the Blackman window and the Hamming window are discussed along with their comparative study.

A. *Hamming window*

The equation for Hamming window sequence can be defined by[12][13][20][27][28],

$$\omega_{Hamm}(n) = \alpha + \beta \cos\left(\frac{2\pi n}{N-1}\right) \quad for \quad -(N-1)/2 \le n \le (N-1)/2$$
$$= 0 \qquad otherwise \qquad ....(1.8)$$

Where,

α, β = coefficient

Now for Hamming window, the value of the coefficients are[14][17][19][20][27],

$$\alpha = 0.54$$

$$\beta = 1 - \alpha = 1 - 0.54 = 0.46$$

Now substituting the values of α and β into eqn(1.8) we get ,

$$\omega_{Hamm}(n) = 0.54 + 0.46 \cos\left(\frac{2\pi n}{N-1}\right) \quad for \quad -(N-1)/2 \le n \le (N-1)/2$$
$$= 0 \quad otherwise$$
....(1.9)

Now the frequency response of the Hamming window can be expressed by[15][16][26][27][28],

$$W_{Hamm}\left(e^{j\omega}\right) = 0.54 \frac{\sin\frac{\omega N}{2}}{\sin\frac{\omega}{2}} + 0.23 \frac{\sin\left(\frac{\omega N}{2} - \frac{\pi N}{(N-1)}\right)}{\sin\left(\frac{\omega}{2} - \frac{\pi}{(N-1)}\right)} + 0.23 \frac{\sin\left(\frac{\omega N}{2} + \frac{\pi N}{(N-1)}\right)}{\sin\left(\frac{\omega}{2} + \frac{\pi}{(N-1)}\right)} \quad ....(1.10)$$

So, from eqn(1.8) through eqn(1.10), a suitable FIR filter can be designed. The Frequency response is also used as the Magnitude response. So with the help of the above equations, the FIR filter with a specified order can be designed with the order variation of N. The coefficients can also be determined with the values of $h_d(n)$.

B. Blackman window

The Blackman window sequence can be defined by[1][16][17][26][27][28],

$$\omega_{Black}(n) = a_0 + a_1 \cos\left(\frac{2\pi n}{N-1}\right) + a_2 \cos\left(\frac{4\pi n}{N-1}\right) \quad for \quad -(N-1)/2 \le n \le (N-1)/2$$
$$= 0 \quad otherwise$$
....(1.11)

Where,

a_0, a_1, a_2 = coefficients

The coefficient a_0, a_1, a_2 can be defined with help of the value of α i.e,

$$a_0 = \frac{1-\alpha}{2}; \qquad a_1 = \frac{1}{2}; \qquad a_2 = \frac{\alpha}{2}$$

Generally, the value of α for Blackman window is static and it is α =0.16. Now, putting the value of α into a_0, a_1, a_2 , we can achieve the actual value of the coeficients for the Blackman window sequences and they are,

$$a_0 = 0.42; \qquad a_1 = 0.5; \qquad a_2 = 0.08$$

Now, substituting the value of a_0, a_1, a_2 into eqn (1.11), we get the actual equation for the Blackman window sequence i.e.[9][12][13][25][26][27],

$$\omega_{Black}(n) = 0.42 + 0.5 \cos\left(\frac{2\pi n}{N-1}\right) + 0.08 \cos\left(\frac{4\pi n}{N-1}\right) \quad for \quad -(N-1)/2 \le n \le (N-1)/2$$
$$= 0 \quad otherwise$$
....(1.12)

Now, with the FIR filter can be designed with the help of these two window technique and this can be best viewed in the magnitude reqsponse plot which will be shown latter.

VI. REALIZATION OF FIR FILTER

Realization of FIR filter is essential to determine the output sequence depending on the previous inputs. There are a number of methods available for the realization of FIR filter. The realization of FIR filter requires the system function of that specied filter. In general, the transfer function can be expressed as[2][3][16][18][23][26][27][28],

$$H(z) = \sum_{n=0}^{\infty} h(n)z^{-n}$$

$$= h(0) + h(1)z^{-1} + h(2)z^{-2} + \ldots + h(N-1)z^{-(N-1)}$$

....(1.13)

As we know $H(z)$ is equal to the ratio of output and input i.e,

$$H(z) = \frac{Y(z)}{X(z)}$$

$$= \sum_{n=0}^{\infty} h(n)z^{-n}$$

....(1.14)

$$= h(0)X(z) + h(1)z^{-1}X(z) + h(2)z^{-2}X(z) +$$

$$\ldots + h(N-1)z^{-(N-1)}X(z)$$

Now to realize the FIR filter using the eqn (1.14), the Transversal structure or simply the Direct form structure can be used. So, using the transversal structure, the eqn(1.14) can be realized as follows[7][8][17][20][27][28]:
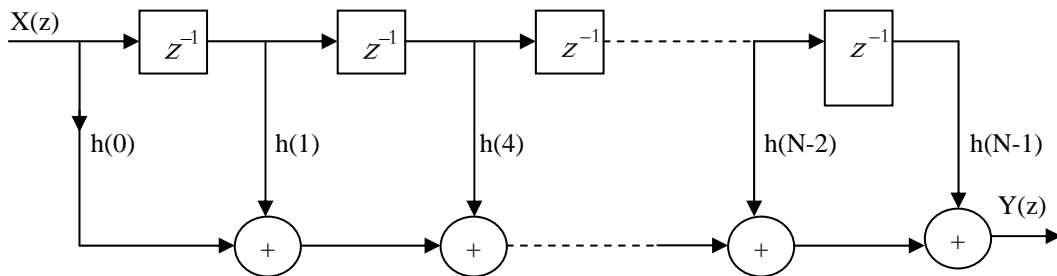

Fig. 1 Transversal structure realization of FIR filter

Now, from the above structure of realization, it is clear that transversal structure requires N multipliers, N-1 adders and N-1 delay elements.

VII. SIMULATION AND DETERMINATION OF WINDOW SEQUENCE AND DESIRED IMPULSE RESPONSE

In FIR filter, whatever may be the type of the filter, the magnitude response must be 1 at the passband and otherwise it will be decreased to zero with or without ripple. So, the desired frequency response or in the other hand the magnitude function can de expressed by,

$$H_d\left(e^{j\omega}\right) = 1 \quad for \quad passband$$

$$= 0 \quad otherwise$$

....(1.15)

Now, recalling the eqn (1.6), it can be shown that the desired impulse response can de derived to,

$$h_d(n) = \frac{\sin n\pi - \sin\frac{\pi}{4}n}{n\pi}$$

....(1.16)

If the the desired impulse response have the symmetry about n=0, it can be obviously said that,

$$h_d(n) = h_d(-n)$$

....(1.17)

Hence, for a FIR filter of order N=17, the desired impulse response can be derived from the eqn (1.16) and given in the Table-1.

Table-1

Desired impulse response $h_d(n)$	Value of $h_d(n)$	Symmetry of $h_d(n)$
$h_d(0)$	0.750	$h_d(0)$
$h_d(1)$	-0.225	$h_d(1)$
$h_d(2)$	-0.159	$h_d(-2)$
$h_d(3)$	-0.075	$h_d(-3)$
$h_d(4)$	0.000	$h_d(-4)$
$h_d(5)$	0.045	$h_d(-5)$
$h_d(6)$	0.053	$h_d(-6)$
$h_d(7)$	0.032	$h_d(-7)$
$h_d(8)$	0.000	$h_d(-8)$

The filter coefficients can be expressed by the following equation,

$$h(n) = h_d(n)\,\omega_{Hamm}(n) \qquad\qquad ....(1.18)$$

a. *Hamming Window*

The filter coefficient $h(n)$ using Hamming window sequence are shown in Table-2.

Table-2

Filter coefficients $h(n)$	Value of coefficient	Symmetry of $h(n)$
$h(0)$	0.7500	$h(0)$
$h(1)$	-0.2169	$h(-1)$
$h(2)$	-0.1375	$h(-2)$
$h(3)$	-0.0537	$h(-3)$
$h(4)$	0.0000	$h(-4)$
$h(5)$	0.0163	$h(-5)$
$h(6)$	0.0113	$h(-6)$
$h(7)$	0.0036	$h(-7)$
$h(8)$	0.0000	$h(-8)$

The simulation result for FIR filter using Hamming window are shown below from Fig.2 to Fig.5.
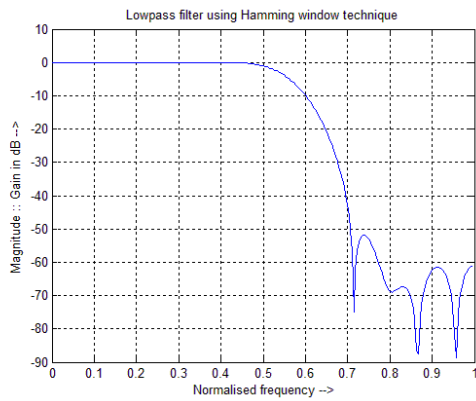
Fig.2 Lowpass FIR filter using Hamming window
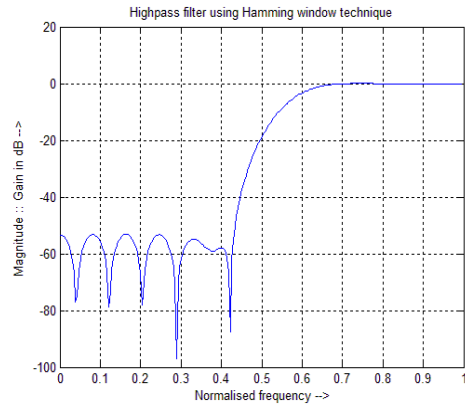

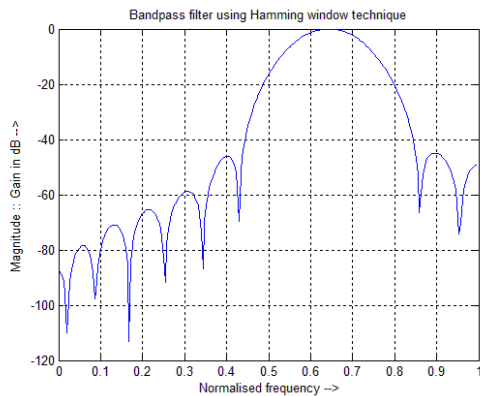Fig.3 Highpass FIR filter using Hamming window

Fig.4 Bandpass FIR filter using Hamming window

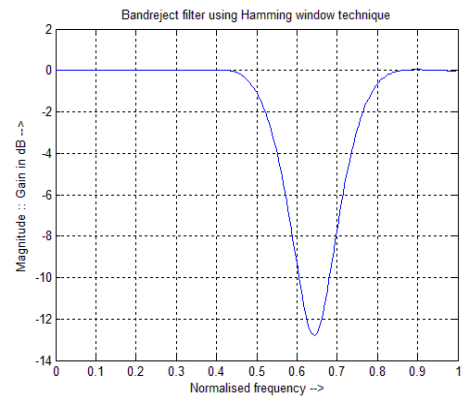Fig.5 Band-reject FIR filter using Hamming window

    b.   *Blackman Window*

The filter coefficient $h(n)$ using Blackman window sequence are shown in Table-3.

Table-3

Filter coefficients $h(n)$	Value of coefficient	Symmetry of $h(n)$
$h(0)$	0.7500	$h(0)$
$h(1)$	-0.2111	$h(-1)$
$h(2)$	-0.1229	$h(-2)$
$h(3)$	-0.0416	$h(-3)$
$h(4)$	0.0000	$h(-4)$
$h(5)$	0.0128	$h(-5)$
$h(6)$	0.0035	$h(-6)$
$h(7)$	0.0004	$h(-7)$
$h(8)$	0.0000	$h(-8)$

The simulation result for FIR filter using Blackman window are shown below from Fig.6 to Fig.9.
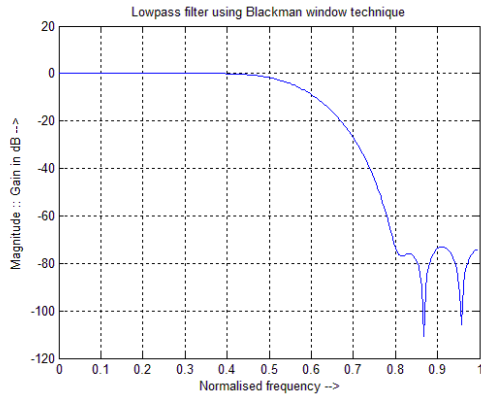
Fig.6 Lowpass FIR filter using Blackman window
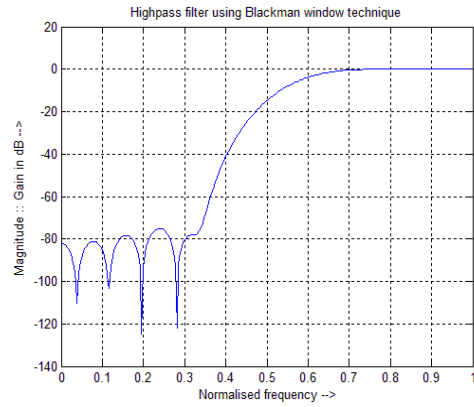

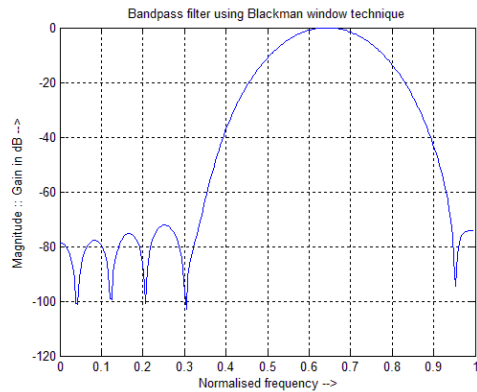Fig.7 Highpass FIR filter using Blackman window

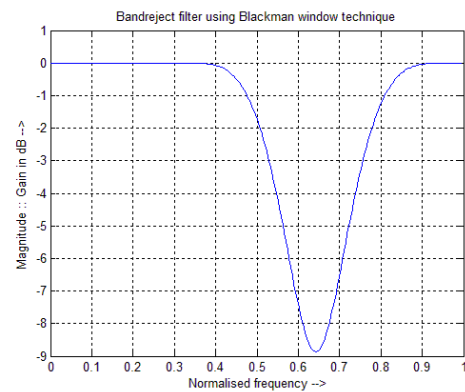Fig.8 Bandpass FIR filter using Blackman window

Fig.9 Band-reject FIR filter using Blackman window

From above eight figures (Fig.2 to Fig.9), we can see that the response of Blackman windoe are more smooth and perfect than that of the Hamming window. So, the Black man window is more perfect and advantagious than that of Hamming window. The advantages of Blackman window over Hamming window are as follows:

1. In window sequence function, the Blackman window have more terms than compared to the Hamming window. More term in calculation indicates more accuracy in result. For this reason, the Result and the simulation both are more accurate in Blackman window.

2. There are an extra cosine term in Blackman function. This extra term in terms reduces the side lobes. In Fig.2 for FIR lowpass filter design, there are four sidelobes exists in case of Hamming window but for same specification and filter order, in Blackman window, there exists only two sidelobes in Fig.6. Reducing sidelobes meaning that the efficiency is increased losing less power. It also helpful as Blackman window also increases the main lobe width to $\dfrac{12\pi}{N}$. The same thing can be observed in other figure also.

3. In Fig.2 we can see that in Hamming window, the peak side lobe is down about 52dB whereas in case of Blackman window, the peak sidelobe is down about 78dB in Fig.6 . So, there is an improvement in Blackman window of 16dB when compared to the Hamming window.

So, it is clear that Blackman window technique is more powerful and perfect than the FIR filter designed with a Hamming window.

VIII. CONCLUSIONS

In this paper, the Realization, design and the simulation are shown for the design of the FIR filter using Blackman window and the Hamming window. Throughout the paper it is also shown the advantages of Blackman window over the Hamming window. The simulations are done using Matlab 7.6 and the desired results are obtained for the madnutude response including the main lobe and the side lobe. The filter coefficients are also calculated for the design of the FIR filter.

REFERENCES

[1] Datar A., Jain A.,Sharma, P.C. ,''Performance of Blackman window famiy in M-channel cosine modulated filter bank for ECG signal'', Multimedia, Signal Processing and Communication Technologies, 2009. IMPACT '09. International, IEEE Conference,Aligarh,ISBN: 978-1-4244-3602-6, pp 98 – 101.

[2] Yu-Chi Tsao, Ken Choi,'' Hardware-Efficient Vlsi Implementation For 3-Parallel Linear-Phase Fir Digital Filter Of Odd Length'', Circuits And Systems (Iscas), 2012 Ieee Conference, Isbn:978-1-4673-0218-0,Pp 998 – 1001.

[3] Soni, V., Shukla P.,Kumar M.,''Application Of Exponential Window To Design A Digital Nonrecursive Fir Filter'', Advanced Communication Technology (Icact), 2011 13th International Conference,Ieee,Isbn: 978-1-4244-8830-8,Pp 1015-1019.

[4] Mehboob, R., Khan, S.A. ; Qamar, R. ,"Fir Filter Design Methodology For Hardware Optimized Implementation", Consumer Electronics, Ieee Transactions,2009, Volume:55 , Issue: 3 ,Pp 1669 – 1673.

[5] Sanal, M.,Kuloor, R.,Sagayaraj, M.J. "Optimized FIR filters for digital pulse compression of biphase codes with low sidelobes", Aerospace Conference, 2013 IEEE,ISBN: 978-1-4673-1812-9, pp-1-9.

[6] Saurabh Singh Rajput, Dr.S.S. Bhadauria, "Implementation of fir filter using efficient window function and its application in filtering a speech signal",International Journal of Electrical, Electronics and Mechanical Controls, Volume 1 Issue 1 November 2012.

[7] Ravinder Kaur, Ashish Raman, Hardev Singh and Jagjit Malhotra, "Design and Implementation of High Speed IIR and FIR Filter using Pipelining", International Journal of Computer Theory and Engineering, Vol. 3, No. 2, April 2011,ISSN: 1793-8201,pp 292-295.

[8] Ramesh .R, Nathiya .R, "Realization of fir filter using modified distributed arithmetic architecture", Signal & Image Processing : An International Journal (SIPIJ) Vol.3, No.1, February 2012,pp 83-94.

[9] S. M. Shamsul Alam , Md. Tariq Hasan, "Performance Analysis of FIR Filter Design by Using Optimal, Blackman Window and Frequency Sampling Methods", International Journal of Electrical & Computer Sciences IJECS-IJENS Vol: 10 No: 01, pp13-18.

[10] Sarita Chouhan, Yogesh Kumar, "Low power designing of FIR filters", International Journal of Advanced Technology & Engineering Research, ISSN NO: 2250-3536 Volume 2, Issue 2, May2012, pp-59-67.

[11] T Ramesh Reddy,Dr. K. Soundara Rajan, "Low Power and Low Area Digital FIR Filter Using Different Multipliers and Adders", International Journal of Engineering Research & Technology (IJERT),Vol. 1 Issue 3, May – 2012,ISSN: 2278-0181,pp 1-4.

[12] Magatha Nayak Bhukya, K. Anjaiah, G. Sravya, P. Nagaraju, "The Design of High Speed FIR Filter using Improved DA Algorithm and it's FPGA Implementation", International Journal of Engineering Trends and Technology- Volume3Issue2- 2012, pp 123-126.

[13] Ram Kumar Soni, Alok Jain, Rajiv Saxena, "An Optimized Design of Non-uniform Filterbank using Blackman Window Family", International Journal of Signal and Image Processing (Vol.1-2010/Iss.1),pp 18-23.

[14] Manoj Garg, Rakesh Kumar Bansal,Savina Bansal, "Reducing Power Dissipation in FIR Filter: An Analysis", Signal Processing: An International Journal (SPIJ), Volume(4): Issue(1), pp 62-67.

[15] Mohamed Al Mahdi Eshtawie,Masuri Bin Othman, "An Algorithm Proposed for FIR Filter Coefficients Representation", World Academy of Science, Engineering and Technology 2 2007,pp 57-63.

[16] Sonika Gupta, Aman Panghal, "Performance Analysis of FIR Filter Design by Using Rectangular, Hanning and Hamming Windows Methods", International Journal of Advanced Research in Computer Science and Software Engineering,Volume 2, Issue 6, June 2012 ISSN: 2277 128X,pp 273-277.

[17] Suhaib Ahmed, "Design Analysis of High Pass FIR Filters Using Hanning, Bartlett and Kaiser Windows", International Journal of Advanced Research in Computer Science and Software Engineering, Volume 2, Issue 11, November 2012 ISSN: 2277 128X,pp 153-160.

Books

[18] B.A. Shanoi, "Introduction to Digital Signal Processing and Filter design", Willey Interscience, 2006.

[19] Li Tan,"Digital Signal Processing-Fundamentals and Applications", Academic Press, Elsavier, ISBN: 978-0-12-374090-8, 2008.

[20] Gerard Blanchet and Maurice Charbit, "Digital Signal and Image Processing using Matlab", ISTE Ltd., © HERMES Science Europe Ltd, 2001,© ISTE Ltd, 2006, ISBN-13: 978-1-905209-13-2,ISBN-10: 1-905209-13-4

[21] J.S. Chitode, "Digital Signal Processing", Technical Publication, Pune, ISBN:9788184314243.

[22] Dag Stranneby, "Digital Signal Processing-DSP & Application", Butterworth-Heinemann,Oxford, ISBN:0750648112, 2001.

[23] Michael Weeks, "Digital Signal Processing Using MATLAB and Wavelets", Infinity Science Press, Hingham, Massachusetts, ISBN: 0-9778582-0-0, 2007.

[24] Taan S. ElAli, "Discrete Systems and Digital Signal Processing with Matlab", CRC Press,ISBN 0-203-487117, 2004.

[25] Bob Meddins, "Introduction to Digital Signal Processing", Essential Electronics Series, Newnes, Butterworth-Heinemann, Oxford, ISBN: 0750650486, 2000.

[26] Proakis, J. G. and Manolakis, D. G. 2007. Digital Signal Processing: Principles, Algorithms, and Applications. Pearson Education Ltd.

[27] P. Ramesh Babu,"Digital Signal Processing", Fourth edition, Scitech Publication(India) Pvt. Ltd, Chennai,2008.

[28] Andreas Antoniou, "Digital Signal Processing : Signals, Systems and Filters", Tata McGraw-Hill Education, ISBN-10: 0070636338, 2006.

Author

Subhadeep Chakraborty, born in 1986, is Assistant Professor in Calcutta Institute of Technology. He received the B.Tech degree from Saroj Mohan Institute of Technology, WBUT,India and M.Tech degree from Kalyani Govt. Engineering College, WBUT, India in Electronics and Communication Engineering in 2008 and 2010.The author has been teaching in Calcutta Institute of Technology for 3 years.He has published a number of Journal papers and research papers in the leading International and National journal. His primary research interest includes Digital Signal Processing, Embedded System and Microprocessor.