Transition Bandwidth Analysis of Infinite Impulse Response Filters

Sujata Prabhakar
Department of Electronics and Communication
UCOE Punjabi University, Patiala

Dr. Amandeep Singh Sappal
Associate Professor
Department of Electronics and Communication
UCOE Punjabi University, Patiala

Abstract - Infinite impulse response (IIR) is a property applying to many linear time-invariant systems. Digital filters are the common example of linear time invariant systems. Systems with this property are called IIR systems or IIR filters. These filters have infinite length of impulse response and are in contrast to finite impulse response (FIR) filters. The basic IIR filters are – Butterworth filters, Chebyshev filters and Elliptic filters. In this paper we are going to discuss some basic theory of these filters and then we will analyze the properties of these filters.

Keywords – IIR filters, FIR filters, Butterworth filters, Chebyshev filters and Elliptic filters.

I. INTRODUCTION

Filters are electronic devices used to modify amplitude and/or phase response of a signal according to their frequency. IIR filters are used for many applications. In [1] IIR filters are used for the suppression of noise in ECG signal. In diagnosis of ECG signal, signal acquisition must be noise free. So the physicians are able to make correct diagnosis on the condition of heart. IIR filters can be used to remove noise present in ECG signal. Adaptive IIR filters [2] can also be used to control active noise. IIR filters can also be used in Image Processing Applications [3]. IIR filters can be implemented on Xilinx system [4] to enhance the computational speed of filters. The speed of computation is greatly increased by implementing a filter on an FPGA, rather than on a conventional DSP processor. Digital IIR filters are designed from the classical analog designs which include the following filter types:

- Butterworth filter
- Chebyshev-I filter
- Chebyshev-II filter, also known as inverse Chebyshev filter
- Elliptic filter, also known as Cauer filter

II. BUTTERWORTH FILTERS

This filter is characterized by the property that the magnitude response is flat both in passband and stopband. The magnitude response [5] of an Nth-order lowpass filter is given by

$$|B(j\omega)|^2 = \frac{1}{1 + \left({}^{j\omega}/_{j\omega_c}\right)^{2N}} \tag{1}$$

Where N is the order and ω_c is the cutoff frequency. The plot of the magnitude response is shown in Fig. 1

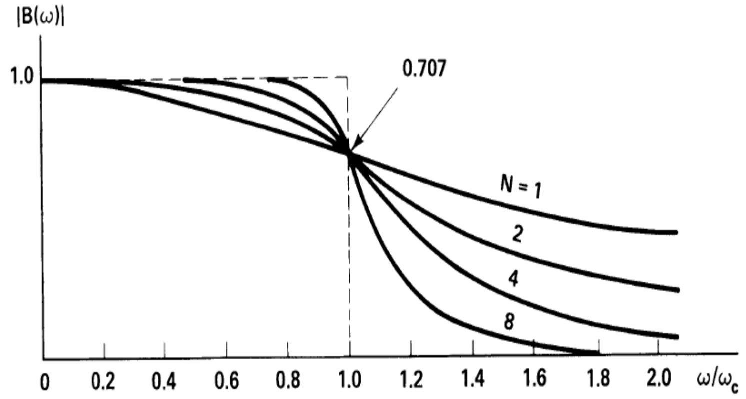

Fig.1 Magnitude Response of a Butterworth Filter

From this plot we can say that at zero frequency, the magnitude of the filter is 1 for all N of the filter. At the cutoff frequency the square of the magnitude of the filter is ½ for all N, which implies a 3 decibel attenuation at cutoff frequency. Also the magnitude response is monotonically decreasing function of frequency. As the order of the filter increases, sharpness of the transition band increases. Ideally as order approaches infinity, the magnitude response approaches an ideal low pass filter.

III. CHEBYSHEV TYPE-I FILTER

Chebyshev-I filters have equiripple response in passband and monotonically decreasing magnitude response in the stopband. The magnitude-squared response of a chebyshev-I filter is

$$|B(j\omega)|^2 = \frac{1}{1+\ \epsilon^2 T_N^2\left(\frac{\omega}{\omega_C}\right)} \qquad (2)$$

Where N is the order of the filter, ϵ is the passband ripple factor, which is related to R_p, and $T_N(x)$ is the Nth-order chebyshev polynomial given by

$$T_N(x) = \begin{cases}\cos(N\ \cos^{-1}(x)), & 0 \le x \le 1\\ \cosh(\cosh^{-1}(x)), & 1 < x < \infty\end{cases} \quad (3)$$

$$where\ \ x = \frac{\omega}{\omega_c}$$

The equiripple response [6] of the chebyshev filter is due to this polynomial$T_N(x)$. For x in the range of 0 to 1 $T_N(x)$ oscillates between –1 and 1. And for $1 < x < \infty$, $T_N(x)$ increases monotonically to infinity. There are two possible shapes of $|B(j\omega)|^2$, one for N odd and one for N even. Note that $x = {}^{\omega}/_{\omega_c}$ is the normalized frequency.

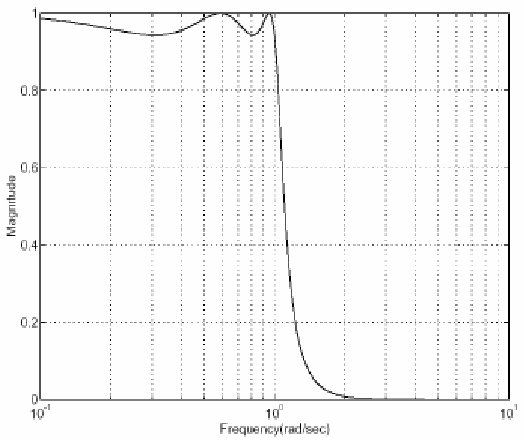

Fig. 2 Magnitude Response of a chebyshev type-I filter with ω_c=1

Compared to the Butterworth filter, a Chebyshev filter can achieve a sharp transition between the passband and stopband with a lower order filter.

IV. CHEBYSHEV TYPE-II FILTER

Chebyshev-II filter is related to Chebyshev-I filter through a simple transformation. It has a monotone passband and an equiripple stopband. If we replace the term $\epsilon^2 T_N^2\left(\frac{\omega}{\omega_C}\right)$ in equation (7) by its reciprocal and also the argument $x = \frac{\omega}{\omega_c}$ by its reciprocal, we obtain the magnitude squared-response of Chebyshev-II as

$$|B(j\omega)|^2 = \frac{1}{1+\ \left[\epsilon^2 T_N^2\left(\frac{\omega_c}{\omega}\right)\right]^{-1}} \qquad (4)$$

We can design a Chebyshev-II filter by first designing the Chebyshev-I filter and then applying this simple transformation. Chebyshev-II filter minimize peak error in the stopband instead of passband. Minimizing peak error in the stopband instead of the passband is an advantage of Chebyshev-II filters over Chebyshev-I filters. Chebyshev-II filter has the same advantage over Butterworth filter as that of Chebyshev-I filter – sharper transition between the passband and the stopband with a lower order filter.

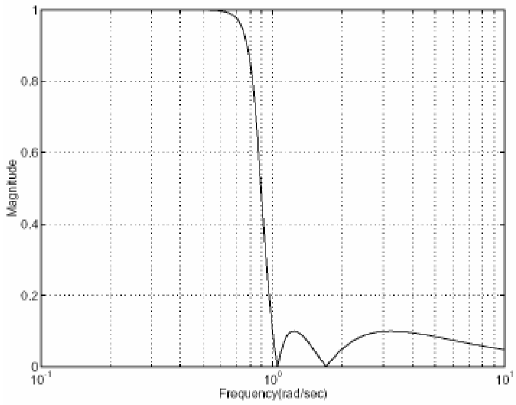

Fig. 3 Magnitude Response of a Chebyshev-II filter with ω_c=1

V. ELLIPTIC FILTERS

These filters exhibit equiripple behavior both in the passband as well as in the stopband. Elliptic filters are very difficult to analyze and, therefore, to design. It is not possible to design these filters using simple tools, and often programs or tables are needed to design them. The magnitude –squared response of elliptic filters is given by

$$|B(j\omega)|^2 = \frac{1}{1 + \epsilon^2 U_N^2\left(\frac{\omega}{\omega_C}\right)} \qquad (5)$$

Where N is the order, ϵ is the passband ripple and $U_N^2(.)$ is the Nthe order Jacobian elliptic function.

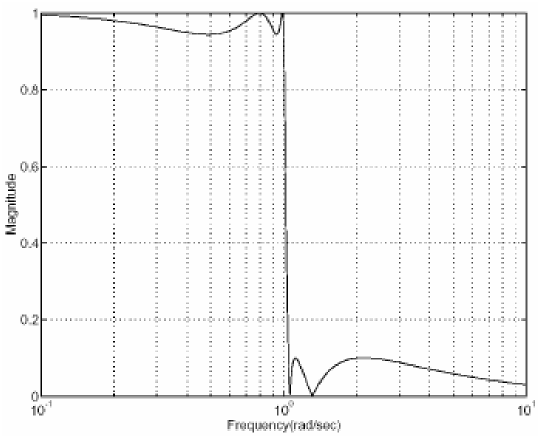

Fig. 4 Magnitude Response of an Elliptic filters with ω_c=1

These filters are similar in magnitude response characteristics to the FIR equiripple filters. Therefore these filters achieve minimum order N for the given specifications or alternatively, achieve the sharpest transition band for the given order N.

VI. RESULT AND ANALYSIS

Basic IIR filters- Butterworth filters, Chebyshev filters and Elliptic filters were discussed in introduction part. These filters were analyzed with their transition band and the order of the filter. A table of transition band of a filter and the Order of the above filters was made as shown below

Table. 1 Effect of Transition band on the order of the filter (Stop band edge frequency is variable)

Sr. no.	Passband Edge frequency (fp)	Stopband Edge Frequency (fs)	Transition Bandwidth	Order of Butterworth	Order of Chebyshev-I	Order of Chebyshev-II	Order of Elliptic
1	4	5	1	19	7	7	4
2	4	4.9	0.9	21	8	8	5
3	4	4.8	0.8	23	8	8	5
4	4	4.7	0.7	27	9	9	5
5	4	4.6	0.6	30	9	9	5
6	4	4.5	0.5	35	10	10	5
7	4	4.4	0.4	44	11	11	5
8	4	4.3	0.3	57	13	13	6
9	4	4.2	0.2	85	16	16	6
10	4	4.1	0.1	167	22	22	7
11	4	4.05	0.05	331	31	31	8

Here in this table pass band edge frequency was made constant and stop band edge frequency was varied to decrease transition bandwidth. As the transition bandwidth was decreased, order of the filter was increasing. But a large increase in order of the filter was there in Butterworth while it is least in case of Elliptic filters. The order of Chebyshev-I and Chebyshev-II remains the same for different values of transition bandwidth. It means these filters can be used interchangeably but the only difference is there in ripples present in passband and stopband. In Chebyshev-I ripples are present in pass band and in Chebyshev-II ripples are present in stop band. The graph of the above table is as shown below.

Now transition bandwidth can be decreased by decreasing passband edge frequency (fp) and making stop band edge frequency (fs) constant. Now the effect of decreasing transition band by varying pass band edge frequency (fp) on various filters is shown below in Table 2

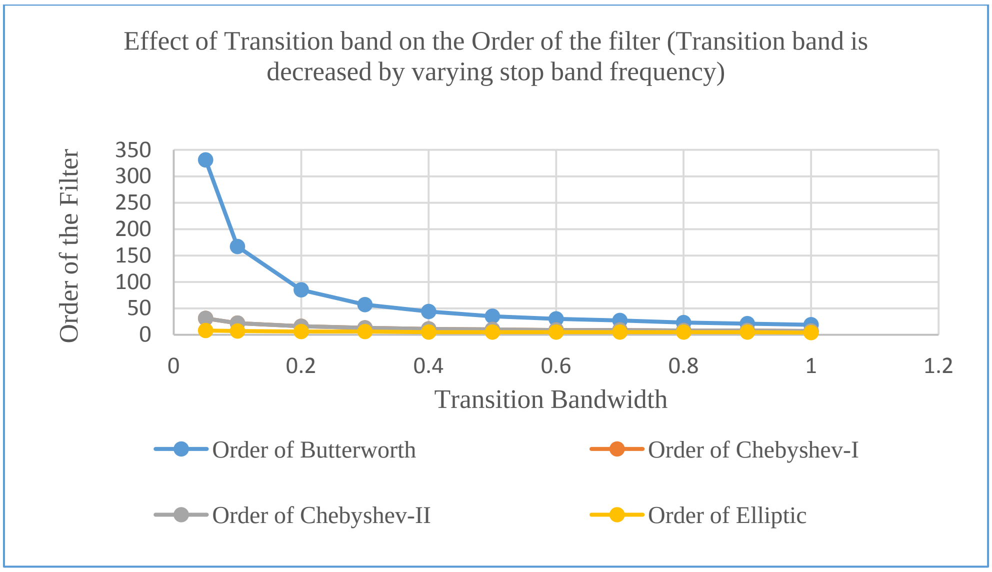

Fig.1 Effect of transition band on the order of the filter (Transition bandwidth is decreased by varying stop band edge frequency)

By comparing Table.2 with Table.1 it can be seen that as the transition band decreases, the order of the elliptic filters almost remains same as that shown in Table.1. The order of the chebyshev filter increases slightly. But the order of the chebyshev-I and chebyshev-II always remains same. But the order of the Butterworth filter increases. The graph of the above Table.2 is also shown below.

Table. 2 Effect of transition band on Order of the filter (Pass band edge frequency is variable)

Sr. no.	Passband Edge frequency (fp)	Stopband Edge frequency (fs)	Transition Bandwidth	Order of Butterworth	Order of Chebyshev-I	Order of Chebyshev-II	Order of Elliptic
1	4	5	1	19	7	7	4
2	4.1	5	0.9	21	8	8	5
3	4.2	5	0.8	24	8	8	5
4	4.3	5	0.7	28	9	9	5
5	4.4	5	0.6	33	10	10	5
6	4.5	5	0.5	39	11	11	5
7	4.6	5	0.4	50	12	12	6
8	4.7	5	0.3	67	14	14	6
9	4.8	5	0.2	101	17	17	6
10	4.9	5	0.1	204	24	24	7
11	4.95	5	0.05	409	34	34	8

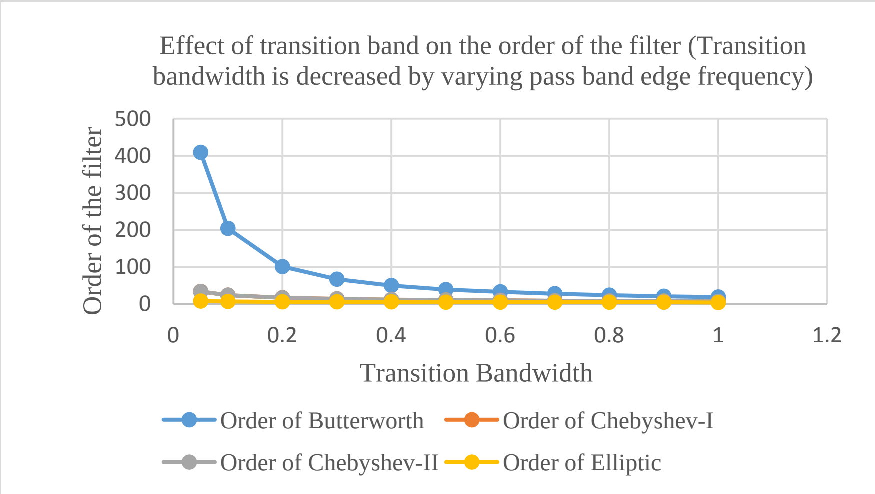

Fig. 2 Effect of transition band on the order of the filter (Transition bandwidth is decreased by varying pass band edge frequency)

Therefore, by comparing both graphs and tables it can be concluded that for the same transition bandwidth Butterworth filter can have different filter order. It can be seen from Table. 1 and Table. 2, as the transition bandwidth decreases difference in the order of the filter increases and the maximum difference is there at 0.05 KHz transition bandwidth. But the Butterworth filters are maximally flat in nature. In case of Chebyshev filter, there is difference in the order of the filter but not as much as there in Butterworth filter. The order of Chebyshev-I and Chebyshev-II always remains same. Chebyshev-I filter has ripples in pass band and Chebyshev-II has ripples in stop band. In case of Elliptic filters, there is almost no change in the order of the filter. In comparison to other filters, Elliptic filters uses lesser order of the filter which means that cost of development of the filter is much lesser than other filters and the order of the filter always remains same. This means that Elliptic filters are robust in nature. But Elliptic filters are equiripple in nature.

Therefore it depends on the requirement of the filter that which filter is to be used. If the cost of development of the filter is the major concern, descending order of the use of the filter is Elliptic filters, Chebyshev-I and Chebyshev-II filters are at the level as they have same order of the filter and then Butterworth filter. If the robustness of the filter is the main concern, then descending order of the use of the filter is Elliptic filters, Chebyshev-I and Chebyshev-II filters are again at same level as they have same order of the filter and then Butterworth filter. If response of the filter is required to be maximally flat, then the descending order of the use of the filter is Butterworth filter, Chebyshev-II filter, Chebyshev-I and Elliptic filters are at same level because they have ripples in pass band.

VII. CONCLUSION

In this paper we studied applications of basic Infinite Impulse Response filters. Theory of four IIR filters – Butterworth, Chebyshev-I, Chebyshev-II and Elliptic filters were studied. Then these filter were analyzed with the transition bandwidth and the order of the filter and it was concluded that it depends on the requirement of the filter that which filter is to be used. If the cost of development of the filter is the major concern, descending order of the use of the filter is Elliptic filters, Chebyshev-I and Chebyshev-II filters are at the level as they have same order of the filter and then Butterworth filter. If the robustness of the filter is the main concern, then descending order of the use of the filter is Elliptic filters, Chebyshev-I and Chebyshev-II filters are again at same level as they have same order of the filter and then Butterworth filter. If response of the filter is required to be maximally flat, then the descending order of the use of the filter is Butterworth filter, Chebyshev-II filter, Chebyshev-I and Elliptic filters are at same level because they have ripples in pass band.

VIII. REFERENCES

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