Design and Realization of Digital FIR Filter using Dolph-Chebysheb Window

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Abstract—In Digital Signal Processing, one of the most important filter type is the FIR filter which can be designed via various methods. Window technique is the most important technique that is used to design the FIR filter. Apart from various window techniques, Dolph-Chebyshev window has the subject of importance for the design of the FIR filter in efficient way. In this paper, the design methodology of the FIR filter using Dolph-Chebyshev window is shown along with the realization of filter and the designing algorithm. The designing program of the FIR filter is simulated in Matlab 7 which shows the satisfactory result. In this paper, the minimization of the side lobes using Dolph-Chebyshev window are shown and can easily be understood that the minimization in side lobes can increase the efficiency and decrease power consumption so that the FIR filter can work more efficiently.

Keywords-FIR filter, Dolph-Chebyshev window, coefficients, impulse function, realization

I. INTRODUCTION

Filtering process is used in the signal processing domain to achieve a desired frequency band from a system as the output by giving some input to it. The device or the system that is used for the filtering operation is called the filter. In Digital Signal Processing, there are mainly two types of filter, Infinite Impulse Response(IIR) filter and the Finite Impulse Response(FIR) filter. The impulse response of the IIR filter is of infinite duration whereas it is of finite duration in case of FIR filter. The fundamentals behind the FIR filter impulse response is that to settle the response value zero after a finite duration and thus obtaining a finite impulse response. The FIR filter requires no feedback path and thus it has no recursion and hence the FIR filter is the nonrecursive filter. The IIR filter can be designed using various methods and algorithms[1][7][8][9][10][28]. The FIR filter can also be designed by applying different techniques and one of the most important method is the Window method[19][20][22][25][27].

In case of the design of the FIR filer, there are various window methods are available in signal procesing. One of the most important window method is the Dolph-Chebyshev window method[2][3][11][21]. In this paper, the design methodology, algorithm and realization of FIR filter with Dolph-Chebyshev window are shown. Whenever an analog filter is designed, it consists of some active and passive components[7][8][9][10]. The analog filter is then converted or mapped to its equivalent digital filter and thus the IIR filter can be obtained which have the infinite impulse response. To obtain the FIR filter, the Window technique is applied here, more specifically the Dolph-Chebyshev Window and hence the impulse response that can be obtained is of finite duration. The Dolph-Chebyshev Window includes the window function which generates the magnitude response with less ripple or noise when compared to other window methods[1][5][8][10][25].

II. FIR FILTER

FIR filter have some advantageous properties for which the designers prefer this over IIR filter. FIR filter have finite duration impulse response and for that the analysis of the FIR filter and the calculation of the coeficients are more efficient with comparison to the IIR filter. The stability is more in FIR filter than the IIR filter. The phase of the FIR filter is also improved than that of the IIR filter. There various FIR filter available and they are[2][3][7][23][24][27],

1. Lowpass filter
2. Highpass filter
3. Bandpass filter
4. Bandstop filter
5. Allpass filter

All the above mentioned filter are also available in the IIR version but for the eficient study, the FIR filters are designed using various techniques from those IIR filter. The advantages of FIR filter over IIR filters are discussed below[7][8][9][27][28],

1. FIR filters are stable than IIR filter.
2. The design of the FIR filter is easier for its linear phase.

3. At the time of implementation on a finite word length digital system, FIR filter are free of limit cycle oscillations.
4. There are various methods are available for designing the FIR filter.
5. FIR filter require no feedback i.e FIR filter has no recursion.
6. In FIR filter, no rounding error are compounded by summed iteration and for that it is inherently stable.
7. Impulse response is finite.

III. Coefficient of FIR Filter

The output of a discrete-time FIR system is described by the weighted sum of previous inputs and the current input but not the previous outputs as the FIR filter does not support the recursion. If the input to the system be x[n] and the output be y[n], the y[n] can be written by following equation[18][22][23][26][27]:

$$y[n] = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2] + \ldots + b_N x[n-N]$$

$$= \sum_{i=0}^{N} b_i x[n-i] \qquad ....(1.4)$$

where,

x[n] = input signal

y[n] = output signal

b_i = filter coefficient

N = Order of filter

The b_i is important in case of FIR filter. It is generally said that it is the filter coefficient but it can also be said that it is the tap weight of the FIR filter. Depending upon the different values of b_i, the magnitude function will be varied and then a different FIR filter can be designed.

IV. Windowing

Windowing incorporates a function called tapering function or window function which states that if some interval is choosen, it returns with finite non-zero value inside that interval and zero value outside that interval. So, if the window with choosen interval is applied on a IIR system, it will obviously return with a finite non-zero value inside that interval producing a FIR system and all other value that are outside the interval will be zero. So, We can view the finite response inside some predefined interval.

Windowing of any waveform causes its fourier transform to develop non-zero values or in other words spectrum leakage, at frequencies other than the angular frequency provided in that waveform. The non-zero value is highest near at the value of angular frequency and decreased and slowly goes to zero when goes further from the specified angular frequency[12][13][16][23][27].

There are various window technique available in signal processing domain for the design of FIR filter and thy are as follows[18][19][20][25][28],

1. Blackman window
2. Hamming window
3. Rectangular window
4. Triangular window or Bartlett window
5. Hanning window
6. B-Spline window
7. Welch window
8. Parzen window
9. Raised cosine window
10. Kaiser window
11. Dolph-Chebyshev window

There are many advantages of Dolph-Chebyshev window for designing the FIR filter. In this paper, the detailing about the design of FIR filter along with the responses.

V. DOLPH-CHEBYSHEV WINDOW

A. Chebyshev Polynomial

The Dolph-Chebyshev window function can be described by the Chebyshev polynomials which was first used by Dolph in 1946.This was first used to solve the problem of the Radio antenna which have the optimal direction characteristics. The Chebyshev polynomial can be defined by[11][15][16][20][23],

$$T_n(x)_{DlCh} = \begin{cases} \cos(n\cos^{-1}(x)) & |x| \leq 1 \\ \cosh(n\cosh^{-1}(x)) & |x| > 1 \end{cases} \qquad \text{….(1.1)}$$

The recurrence relation that follows immediately from the equation(1.1) are,

$$\begin{aligned} &T_0(x) = 1 \qquad T_1(x) = 1 \\ &T_n(x) = 2xT_{n-1}(x) - T_{n-2}(x) \qquad n \geq 2 \end{aligned} \qquad \text{….(1.2)}$$

Where $T_n(x)$ is the nth order polynomial of x. The sign magnitude of $T_n(x)$ depends upon the sign magnitude of x. $T_n(x)$ will be greater than 1 if the value of x will be greater than 1. For large value of x, $T_n(x)$ becomes with the value that is $T_n(x) \approx 2^{n-1}x^n$ [13][14][17][24][27].

B. Chebyshev Window Function

The Dolph-Chebyshev window $w_0(n)$ can be described by the following equation[19][21][22][26][28],

$$w_0(n)_{DlCh} = \frac{1}{N}\sum_{k=0}^{N-1} W_0(k).e^{i2\pi kn/N} \qquad \text{….(1.3)}$$

Where,

$W_0(k)$ =Fourier coefficient

The fourier coefficient $W_0(k)$ can be derived by the Fourier transform of $w_0(n)$ and hence can be represented by [19][20][23][26][27][28],

$$W_0(k)_{DlCh} = \frac{\cos\left\{N\cos^{-1}\left[\beta\cos\left(\frac{\pi k}{N}\right)\right]\right\}}{\cosh[\cosh^{-1}(\beta)]} \qquad \text{….(1.4)}$$

Where,

β =Fixed valued parameter for Dolph-Chebyshev window function and can be representated by[21][24][26],

$$\beta_{DlCh} = \cosh\left[\frac{1}{N}\cosh^{-1}\left(10^{\alpha}\right)\right] \qquad \text{….(1.5)}$$

Where,

α = Parameter to set the Chebyshev norm of the sidelobes at -20Db and can be represented as,

$$\alpha = \cosh\left(\frac{1}{N}\cosh^{-1}\left(10^{\frac{A}{-20}}\right)\right) \qquad \text{….(1.6)}$$

Where,

A =Required sidelobe attenuation.

So, from equation(1.1) to equation(1.6), a suitable FIR filter can be designed with the proper selection of the designing parameter as well as the order of the filter N.

VI. REALIZATION OF FIR FILTER

There are a number of methods available for the realization of the FIR filter. From the realization, it can be determined and understood that the present output sequence is dependent upon the previous and present input sequences and not on the previous output sequences as FIR filter does not require the feedback path. Generally the transfer function can be described by[3][7][8][9][10][27][28],

$$\begin{aligned} H(z) &= \sum_{n=0}^{\infty} h(n) z^{-n} \\ &= h(0) + h(1) z^{-1} + h(2) z^{-2} + \ldots + h(N-1) z^{-(N-1)} \end{aligned} \qquad \text{....(1.7)}$$

As we know $H(z)$ is equal to the ratio of output and input i.e,

$$\begin{aligned} H(z) &= \frac{Y(z)}{X(z)} \\ &= \sum_{n=0}^{\infty} h(n) z^{-n} \\ &= h(0) X(z) + h(1) z^{-1} X(z) + h(2) z^{-2} X(z) + \\ &\ldots + h(N-1) z^{-(N-1)} X(z) \end{aligned} \qquad \text{....(1.8)}$$

Now to realize the FIR filter using the eqn (1.8), the Direct form can be used. So, using the Direct form, the eqn(1.8) can be realized as follows[24][25][26][27][28],

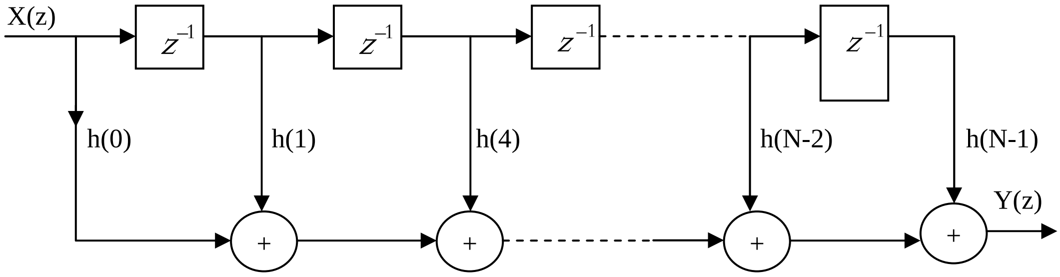

Fig. 1 Direct Form realization of FIR filter

From the above structure of realization, it is clear that Direct form structure requires N multipliers, N-1 adders and N-1 delay elements.

VII. SIMULATION RESULTS

A. Response Parameters

In FIR filter, the magnitude response must be 1 at the passband and otherwise it will be decreased to zero with or without ripple. So, the desired frequency response or in the other hand the magnitude function can de expressed by[26][27][28],

$$\begin{aligned} H_d\left(e^{j\omega}\right) &= 1 \qquad \textit{for} \quad \textit{passband} \\ &= 0 \qquad \textit{otherwise} \end{aligned} \qquad \text{....(1.9)}$$

The desired impulse response can be expressed by,

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d\left(e^{j\omega}\right) e^{j\omega n} d\omega \qquad \text{....(1.10)}$$

The z-transform of $h_d(n)$ is given by,

$$H(z) = \sum_{n=-\infty}^{\infty} h_d(n) z^{-n} \qquad \text{....(1.7)}$$

B. Dolph-Chebyshev Window

The simulation of the Dolph-Chebyshev window that is the plotting is shown below:

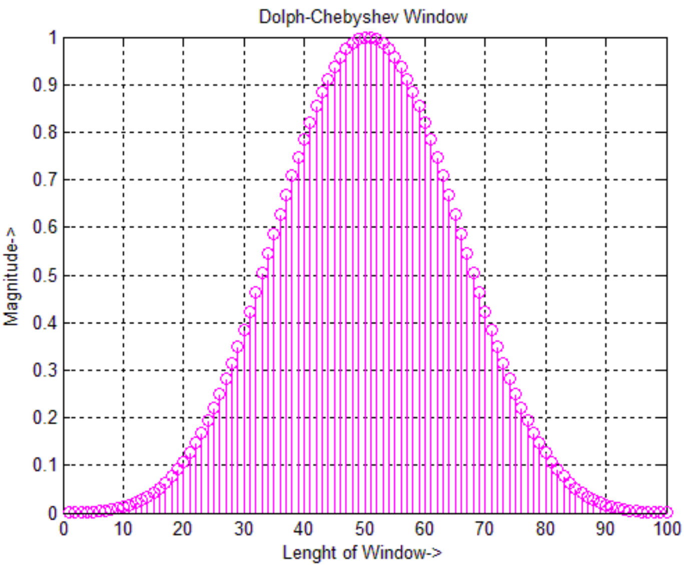

Fig. 2 Dolph-Chebyshev Window for L=100, r=100dB

C. Simulation of FIR Filters

The simulation and their output of the magnitude response, phase response and the impulse responses of the FIR filter by Dolph-Chebyshev window are shown in the figures from Fig. 3 to Fig. 11.

1) Highpass Filter (Order=25)

i. Magnitude response

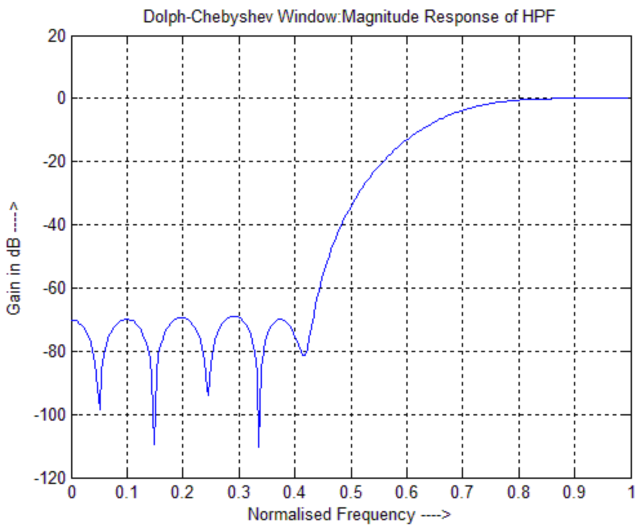

Fig. 3 Magnitude response of FIR Highpass filter using Dolph-Chebyshev window

ii. Phase response

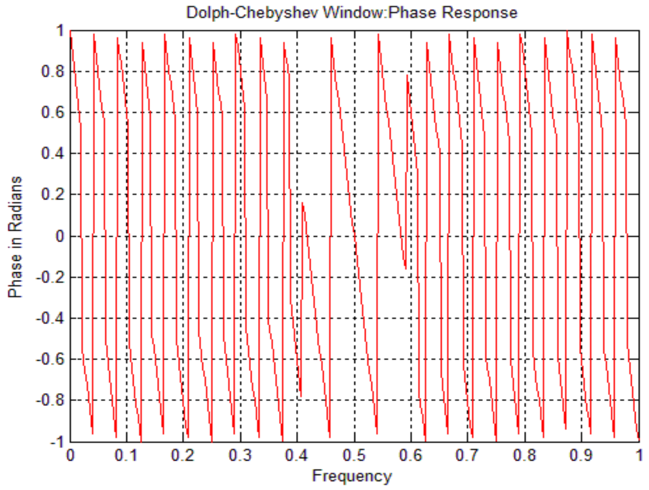

Fig. 4 Phase response of FIR Highpass filter using Dolph-Chebyshev window

iii. Impulse response

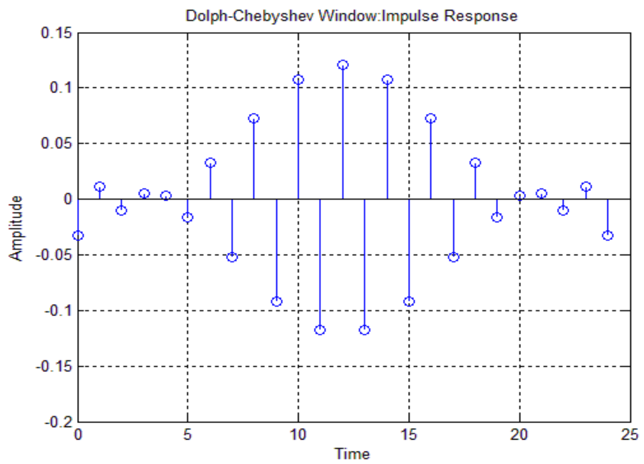

Fig. 5 Impulse response of FIR Highpass filter using Dolph-Chebyshev window

2) Bandpass Filter (Order=25)

i. Magnitude response

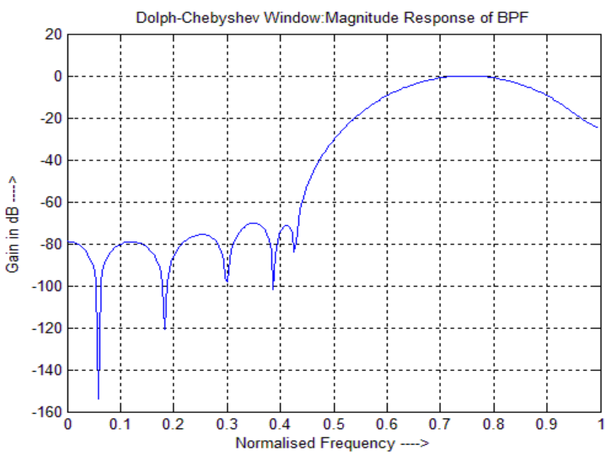

Fig. 6 Magnitude response of FIR Bandpass filter using Dolph-Chebyshev window

ii. Phase response

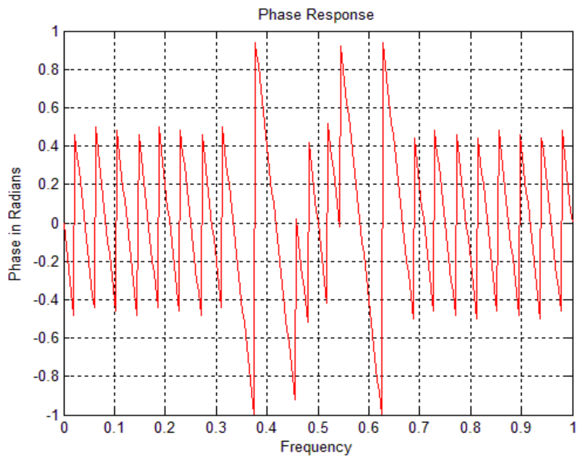

Fig. 7 Phase response of FIR Bandpass filter using Dolph-Chebyshev window

iii. Impulse response

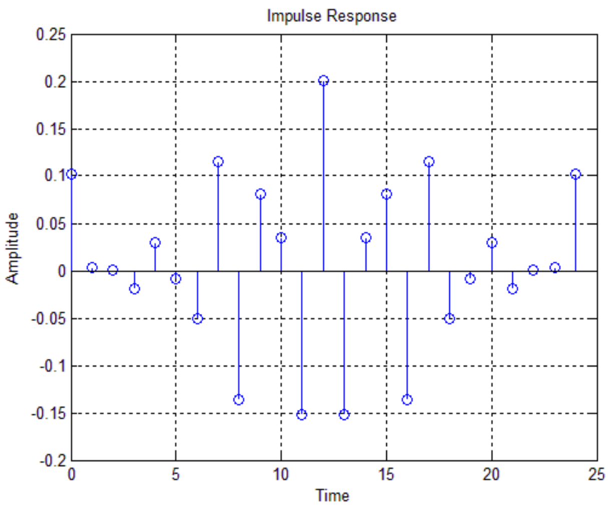

Fig. 8 Impulse response of FIR Bandpass filter using Dolph-Chebyshev window

3) *Bandstop Filter(Order=25)*

i. Magnitude response

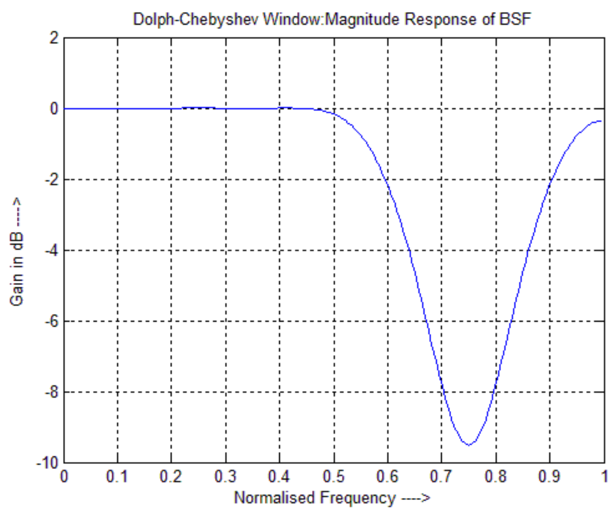

Fig. 9 Magnitude response of FIR Bandstop filter using Dolph-Chebyshev window

ii. Phase response

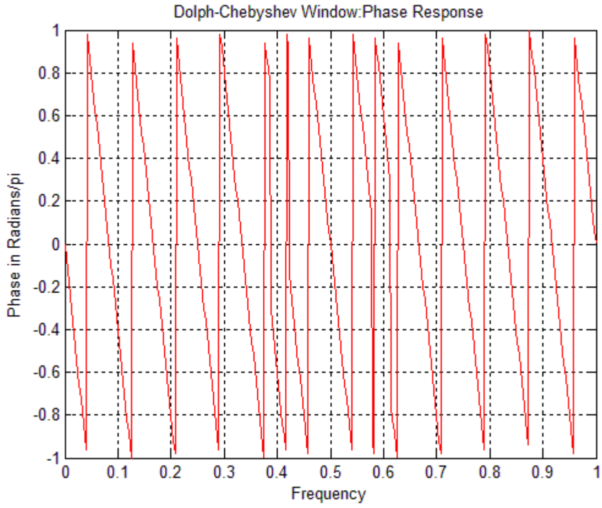

Fig. 10 Phase response of FIR Bandstop filter using Dolph-Chebyshev window

iii. Impulse response

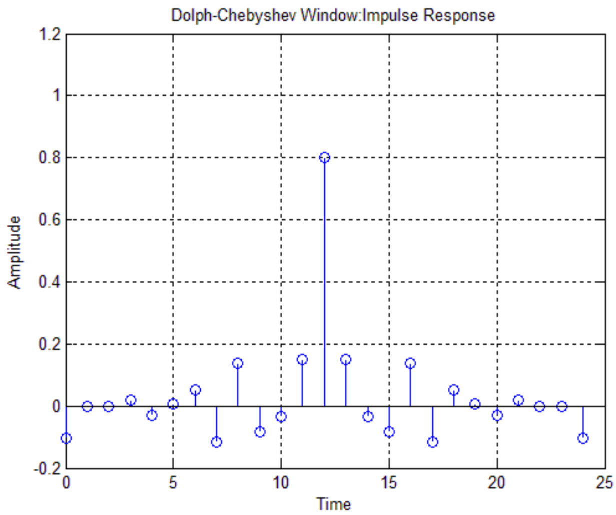

Fig. 11 Impulse response of FIR Bandstop filter using Dolph-Chebyshev window

D. Determination of Function and Response value

1) Highpass Filter

Table - I

Order of Filter	FIR Filter	Response/Function name	Value
25	Highpass Filter	Desired Shifted Response	-0.0261 0.0244 -0.0187 0.0088 0.0050 -0.0219 0.0409 -0.0605 0.0794 -0.0960 0.1089 -0.1172 0.1200 -0.1172 0.1089 -0.0960 0.0794 -0.0605 0.0409 -0.0219 0.0050 0.0088 -0.0187 0.0244 -0.0261
		Window Function	7.6994 0.7853 0.8204 0.8530 0.8829 0.9098 0.9337 0.9542 0.9712 0.9846 0.9942 1.0000 1.0019 1.0000 0.9942 0.9846 0.9712 0.9542 0.9337 0.9098 0.8829 0.8530 0.8204 0.7853 7.6994
		Impulse Response	-0.0323 0.0112 -0.0100 0.0053 0.0034 -0.0164 0.0331 -0.0525 0.0727 -0.0916 0.1070 -0.1172 0.1207 -0.1172 0.1070 -0.0916 0.0727 -0.0525 0.0331 -0.0164 0.0034 0.0053 -0.0100 0.0112 -0.0323

2) Bandpass Filter

Table - II

Order of Filter	FIR Filter	Response/Function name	Value
25	Bandpass Filter	Desired Shifted Response	0.0133 0.0044 0.0000 -0.0218 0.0341 -0.0092 -0.0541 0.1211 -0.1407 0.0827 0.0351 -0.1516 0.2000 -0.1516 0.0351 0.0827 -0.1407 0.1211 -0.0541 -0.0092 0.0341 -0.0218 0.0000 0.0044 0.0133
		Window Function	7.6994 0.7853 0.8204 0.8530 0.8829 0.9098 0.9337 0.9542 0.9712 0.9846 0.9942 1.0000 1.0019 1.0000 0.9942 0.9846 0.9712 0.9542 0.9337 0.9098 0.8829 0.8530 0.8204 0.7853 7.6994
		Impulse Response	0.1022 0.0035 0.0000 -0.0186 0.0301 -0.0084 -0.0505 0.1155 -0.1367 0.0814 0.0349 -0.1516 0.2004 -0.1516 0.0349 0.0814 -0.1367 0.1155 -0.0505 -0.0084 0.0301 -0.0186 0.0000 0.0035 0.1022

3) Bandstop Filter

Table - III

Order of Filter	FIR Filter	Response/Function name	Value
25	Bandstop Filter	Desired Shifted Response	-0.0133 -0.0044 -0.0000 0.0218 -0.0341 0.0092 0.0541 -0.1211 0.1407 -0.0827 -0.0351 0.1516 0.8000 0.1516 -0.0351 -0.0827 0.1407 -0.1211 0.0541 0.0092 -0.0341 0.0218 -0.0000 -0.0044 -0.0133
		Window Function	7.6994 0.7853 0.8204 0.8530 0.8829 0.9098 0.9337 0.9542 0.9712 0.9846 0.9942 1.0000 1.0019 1.0000 0.9942 0.9846 0.9712 0.9542 0.9337 0.9098 0.8829 0.8530 0.8204 0.7853 7.6994
		Impulse Response	-0.1022 -0.0035 -0.0000 0.0186 -0.0301 0.0084 0.0505 -0.1155 0.1367 -0.0814 -0.0349 0.1516 0.8016 0.1516 -0.0349 -0.0814 0.1367 -0.1155 0.0505 0.0084 -0.0301 0.0186 -0.0000 -0.0035 -0.1022

VIII. CONCLUSION

The value of the desired shifted response, window function and the impulse response are calculated in Matlab 7.6 and shown above from Table-I to Table-III and achieved the satisfactory result for designing the FIR filter using Dolph-Chebyshev window. The simulation for the required responses discussed above are simulated also in Matlab 7.6 and shown in the figure from Fig. 3 to Fig. 11. The Dolph-Chebyshev window of length 100 is also shown in the Fig. 2. So, from the above simulation and the calculated results, it can be said that a FIR filter can be suitably designed by using Dolph-Chebyshev window.

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