Analysis of Fuzzy Fault Tree using Intuitionstic Fuzzy Numbers

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Abstract
In generally fuzzy seats are used to analyses the system reliability. In present paper we have presented a new approach to evaluate the reliability of fuzzy fault tree using Intuitionstic Fuzzy Numbers. In application of Intuitionstic Fuzzy fault tree analyses, we have introduced a new distance methods between two interwal estimates of reliability with truth values. Using this methods, the importance index is calculated and compared with the weighted index.

KEYWORDS: Intuitionstic Fuzzy Numbers, Fault Tree Model.

1. Introduction

The concept of fault tree analysis (FTA) was developed in 1962 at Bell telephone laboratories. FTA is now widely used in many fields, such as in nuclear reactor, chemical and aviation industries. Fault tree analysis (FTA) is a logical and diagrammatic method for evaluating system reliability. It is logical approach for systematically quantifying the possibility of abnormal system event. Starting from the top event the fault tree method employs Boolean algebra and logical modeling to represent the relations among various failure events at different levels of system decomposition. FTA can be a qualitative evaluation or quantitative analysis. However, current fault tree analysis still cannot be performed functionally without facing imprecise failure and improper modeling problems. FTA is now widely used in many fields such as in the nuclear reactor and chemical industries.

The reliability of a system is the probability that the system will perform a specified function satisfactorily during some interval of time under specified operating conditions. Traditionally, the reliability of a system behaviour is fully characterized in the context of probability measures, and the outcome of the top event is certain and precise as long as the assignment of basic events are descent from reliable information. However in real life systems, the information may be inaccurate or might have linguistic representation. In such cases the estimation of precise values of probability becomes very difficult. In order to handle this situation, fuzzy approach is used to evaluate the failure rate status. Fuzzy fault tree analysis has been used by several researchers[5,6,7,11,12] and Singer[12] proposed a method using fuzzy numbers to represent the relative frequencies of the basic events. He used possibilistic AND, OR and NEG operators to construct possible fault tree.

2. Basic Notions and Definitions of Intuitionistic Fuzzy Sets (IFS$_s$):-

Fuzzy set theory was first introduced by Zadeh in 1965[3]. Let X be universe of discourse defined by $X = \{x_1, x_2,...,x_n\}$. The grade of membership of an element $x_i \in X$ in a fuzzy set is represented by real value between 0 and 1. It does indicate the evidence for $x_i \in X$, but does not indicate the evidence against $x_i \in X$. Atanassov in 1984[1,2] presented the concept of IFS, and pointed out that this single value combines the evidence for $x_i \in X$ and the evidence against $x_i \in X$. An IFS $\widetilde{A}$ in X is characterized by a membership function $\mu_{\tilde{A}}(x)$ and a non membership function $\nu_{\tilde{A}}(x)$. Here $\mu_{\tilde{A}}(x)$ and $\nu_{\tilde{A}}(x)$ are associated with each point in X, a real number [0 1] with the value of $\mu_{\tilde{A}}(x)$ and $\nu_{\tilde{A}}(x)$ at x representing the grade of membership and non membership of x in$\tilde{A}$.

Thus closure the value of $\mu_{\tilde{A}}(x)$ to unity and the value of $\nu_{\tilde{A}}(x)$ to zero; higher the grade of membership and lower the grade of non-membership of X. When $\tilde{A}$ is a crisp set, its membership function (non-membership) can take only two values zero and 1. If $\mu_{\tilde{A}}(x)$ =1 and $\nu_{\tilde{A}}(x) = 0,$ the element x belongs to $\widetilde{A}$. Similarly if $\mu_{\tilde{A}}(x)$ =0 and $\nu_{\tilde{A}}(x) = 1,$ the element does not belongs to $\widetilde{A}$.

An IFS becomes a fuzzy set $\widetilde{A}$ when $\nu_{\tilde{A}}(x) = 0$ but $\mu_{\tilde{A}}(x) \in [0\ \ 1], \forall\, x \in \widetilde{A}.$

2.1 Definition of Intuitionistic Fuzzy Set: - Let E be a fixed set. An Intuitionistic fuzzy set $\tilde{A}$ of E is an object having the form $\tilde{A} = \{< x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x) >: x \in E\}$

Where the functions

$\mu_A: E \rightarrow [0\ 1]$ and $\nu_A: E \rightarrow [0\ 1]$ define respectively, the degree of membership and the degree of non-membership of the element $x \in E$ to the set A, which is a subset of E and for every $x \in E,\ 0 \leq \mu_A(x) + \nu_A(x) \leq 1.$

When the universe of discourse E is discrete, an IFS $\widetilde{A}$ can be written as

$$\tilde{A} = \sum_{i=1}^{n} [\mu_A(x),\ \ 1 - \nu_A\,(x)]/x, \forall\, x_i \in E$$

An IFS $\widetilde{A}$ with continuous universe of discourse E can be written as

$$\tilde{A} = \int_E\ [\mu_A(x), 1 - \nu_A(x)]/x, \forall x_i \in E$$

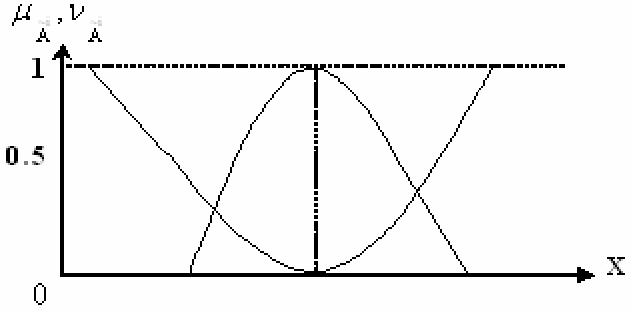


Fig. 1 Membership and non-membership functions of $\check{A}$

2.2 Triangular Intuitionistic Fuzzy Numbers (TIFN):-

The TIFN $\tilde{A}$ is an Intuitionistic Fuzzy number ($\tilde{A}$) is an Intuitionistic Fuzzy set in R with five real numbers $(a_1, a_2, a_3, a', a'')\ with\ (a' \leq\ a_1 \leq a_2 \leq a_3 \leq a'')$ and two triangular functions

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & for\ a_1 \leq x \leq a_2 \\ \frac{a_3-x}{a_3-a_2}, & for\ a_2 \leq x \leq a_3 \\ 0, & otherwise \end{cases}$$

$$\nu_{\tilde{A}}(x) = \begin{cases} \frac{a_2-x}{a_2-a'}, & for\ a' \leq x \leq a_2 \\ \frac{x-a_2}{a''-a_2}, & for\ a_2 \leq x \leq a'' \\ 1, & otherwise \end{cases}$$

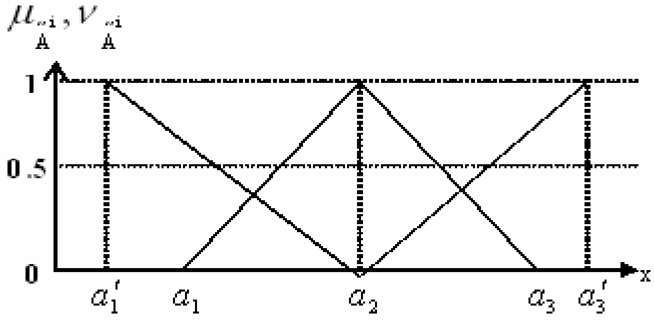


Fig. 2 Membership and non-membership functions of TIFN

2.3 Existing Measuring Methods of distance between IFSs: - For two Intuitionistic fuzzy sets of X are denoted by, with truth-membership t_A, t_B and false-membership f_A, f_B, respectively. Atannassove suggested the distance as follows:

The Hamming distance $\hat{l}\,(A, B)$ is given by

$$\hat{l}\,(A,B)=\frac{1}{2n}\sum_{i=1}^{n}(|t_A(x_i)-t_B(x_i)|+|f_A(x_i)-f_B(x_i)|) \quad -------(1)$$

The Euclidean distance $\hat{q}\,(A,B)$ is given by

$$\hat{q}(A,B)=\sqrt{\frac{1}{2n}\sum_{i=1}^{n}(|t_A(x_i)-t_B(x_i)|^2+|f_A(x_i)-f_B(x_i)|^2\,)} \quad -------(2$$

Szmidt and Kacprzyk gave a geometrical interprepatian of IFSs, and then they proposed corresponding modified distances union took account the three parameters of Intuitionistic fuzzy sets. The definitions of distance given by them are as follows:

The Hamming distance $l''\,(A,B)$

$$l''\,(A,B)=\frac{1}{2n}\sum_{i=1}^{n}(|t_A(x_i)-t_B(x_i)|+|f_A(x_i)-f_B(x_i)|\;+|\pi_A(x_i)-\pi_B(x_i)|) \quad ------(3)$$

The Euclidean distance $q''(A,B)$ is given by

$$q''(A,B)=\sqrt{\frac{1}{2n}\sum_{i=1}^{n}(|t_A(x_i)-t_B(x_i)|^2+|f_A(x_i)-f_B(x_i)|^2+|\pi_A(x_i)-\pi_B(x_i)|^2\,)} \quad -------(4$$

Based on Hausdroff metric, Grzegorzewski proposed another group of distances.

The Hamming distance $l_n\,(A,B)$ is given

$$l_n\,(A,B)=\frac{1}{n}\sum_{i=1}^{n}\max\,(|t_A(x_i)-t_B(x_i)|,|f_A(x_i)-f_B(x_i)|) \quad ------(5)$$

The Euclidean distance $q_n\,(A,B)$ is given by

$$q_n\,(A,B)=\sqrt{\frac{1}{n}\sum_{i=1}^{n}\max\,(|t_A(x_i)-t_B(x_i)|^2,|f_A(x_i)-f_B(x_i)|^2)} \quad ------(6$$

Lu and Wang proposed a distance measurement

$$d_n\,(A,B)=\frac{1}{n}\sum_{i=1}^{n}(\frac{|s_A(x_i)-s_B(x_i)|}{4}+\frac{|t_A(x_i)-t_B(x_i)|}{4}+\frac{|f_A(x_i)-f_B(x_i)|}{4}) \quad ------(7)$$

Where

$$s_A(x_i)=|t_A(x_i)-f_A(x_i)|$$

$$s_B(x_i)=|t_B(x_i)-f_B(x_i)|$$

New Distance between IFSs:-

As we know, the distance suggested by Atannassove[1,2] are the orthogonal projections of the distance presented by Szmidt[10] and Kacprzyk. In this present paper, we first correlate the distance suggested by Atannassove[1,2] and Grzegorzewski,[14] and then propose a new group of distance to evaluate the fuzzy reliability of the system.

Besides the Hamming and Euclidean distance, for two ordinary fuzzy sets A, B of X, with membership function t_A, t_B , then normalized Minkowski's distance can be defined as follows:

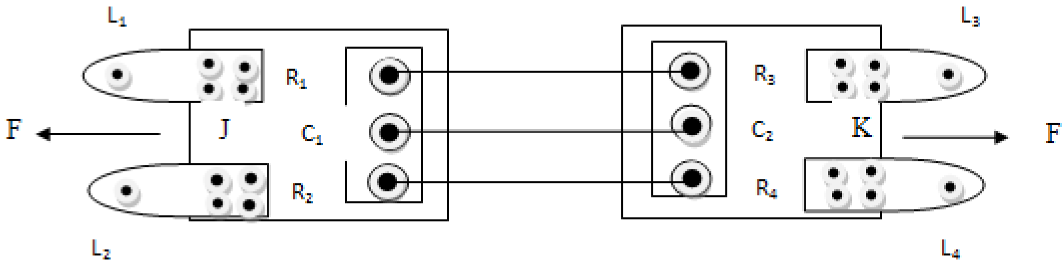
$$l_m\,(A,B)=\left(\sum_{i=1}^{m}\frac{|t_A(x_i)-t_B(x_i)|^p}{n}\right)^{1/p} \quad ----(8)$$

Naturally, we want to extend it to IFS,

$$l_1\,(A,B)=\left(\sum_{i=1}^{m}\frac{|t_A(x_i)-t_B(x_i)|^p+|f_A(x_i)-f_B(x_i)|^p}{2n}\right) ----(9)$$

It is easy to verify that equation (9) will generate to equation (1) and (2) when p=1, 2 respectively.

3. Model Description:-

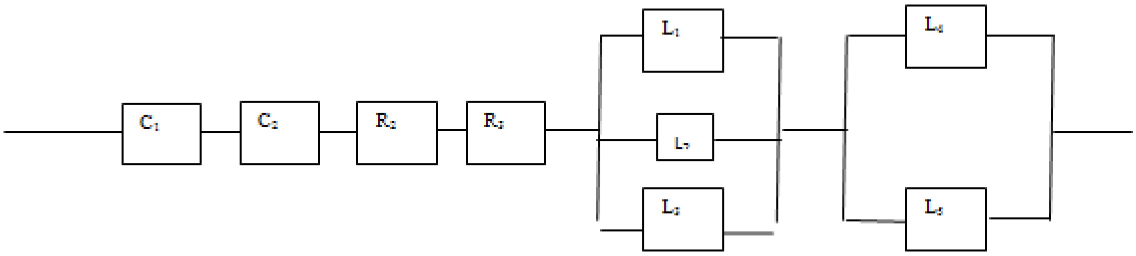


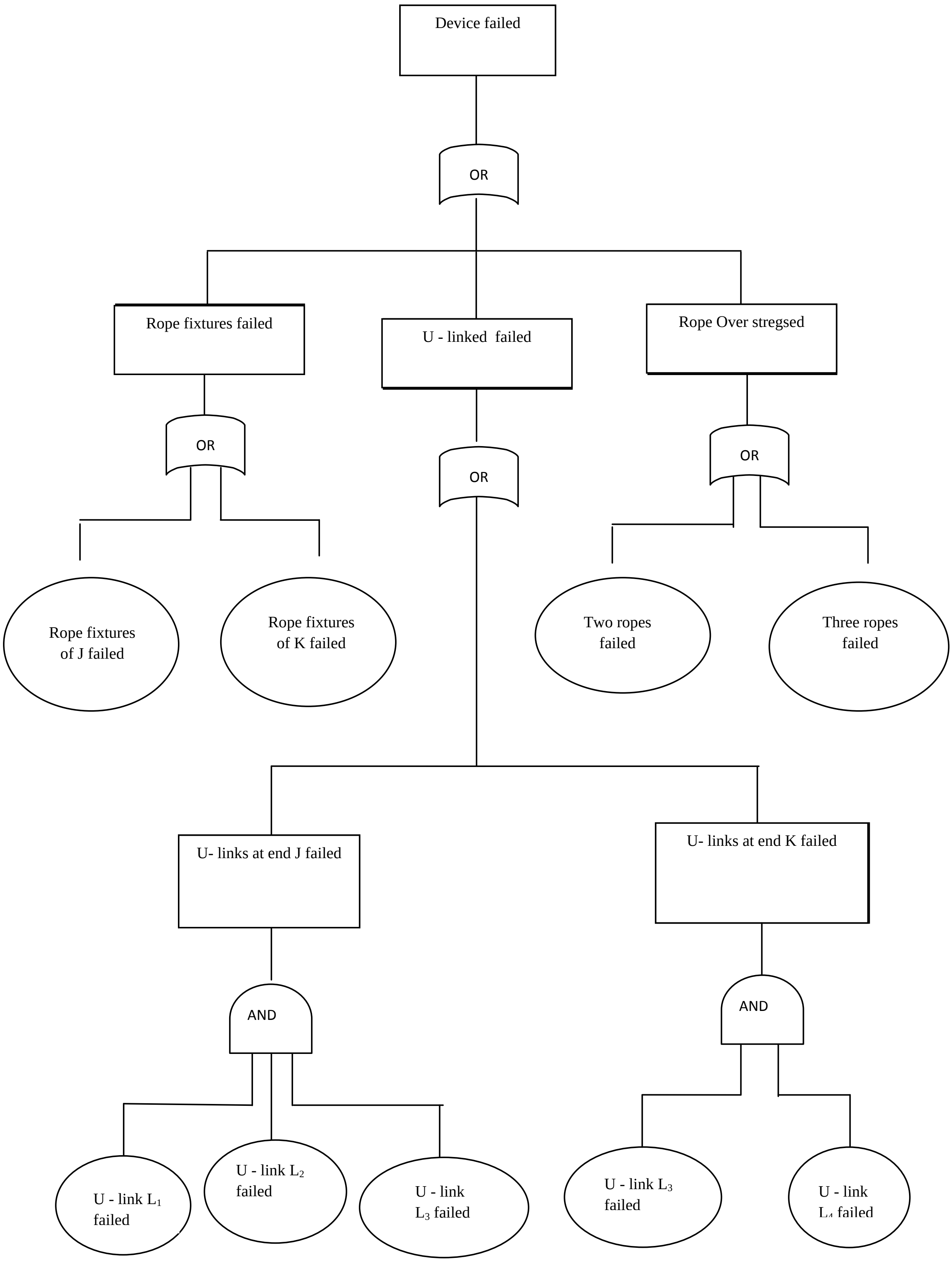
This model shows a device designed to heavy weights. Three steel ropes are connected to two heavy end plates J and K. Each end plate has two U-links bolted to it and each U-link is held by four bolts. When the applied load exceeds the design load, failure of the device may occur due to one or more of the following causes:

(a) Failure of the bolts holding each U-link, since there are four bolts holding each link, the failure of each of there bolts may be denoted by R_1, R_2, R_3,R_4.

(b) Any two of the steel ropes, or all the three ropes may fail due to over stressing. Lets R_2 denotes the failure of two ropes and R_3 denotes the failure of all the three ropes.

(c) The fixtures of the ropes to each of the end plates may fail. Denote these by C_1 and C_2 referring to the left and right end plate fixtures respectively.




Device failed
OR
Rope fixtures failed
U - linked failed
Rope Over stregsed
OR
OR
OR
Rope fixtures of J failed
Rope fixtures of K failed
Two ropes failed
Three ropes failed
U- links at end J failed
U- links at end K failed
AND
AND
U - link L1 failed
U - link L2 failed
U - link L3 failed
U - link L3 failed
U - link L4 failed

4. Numerical Computations:-

According to arithmetic operations of triangle Intuitionistic fuzzy sets the failure range of system failure shown in the model" can be described as:

$$P_T = [\{1-\left(1-P_{A_1}.P_{A_2}\right)\left(1-P_{B_1}.P_{B_2}\right)\left(1-P_C\right)\}\times\{1-P_Z\}$$
$$\times\{\left(1-P_{D_1}.P_{D_2}\right)\left(1-P_{E_1}.P_{E_2}\right)\left(1-P_{F_1}.P_{F_2}\right)\}$$
$$\times\{\left(1-P_{G_1}\right)\left(1-P_{G_2}\right)\left(1-P_{G_3}\right)\}$$

= {1(-) [(0.992, 0.995, 0.998); 0.8], [(0.99, 0.995, 1.0); 0.9]
(×) [(0.992, 0.993, 0.994); 0.8], [(0.991, 0.993, 0.997); 0.9]
(×) [(1, 1, 1); 0.6], [(1, 1, 1); 0.7]
(×) [(0.991, 0.992, 0.993); 0.7], [(0.99, 0.992, 0.994); 0.8]
(×) [(1, 1, 1); 0.6], [(1, 1, 1); 0.8]
(×) [(0.99974, 0.9999, 0.99995); 0.8], [(0.9997, 0.9999, 0.99999); 0.9]
(×) [(0.992, 0.994, 0.995); 0.8], [(0.99, 0.994, 0.997); 0.8]
(×) [(0.991, 0.993, 0.993); 1.0], [(0.991, 0.993, 0.993); 1.0]
(×) [(0.994, 0.995, 0.996); 0.8], [(0.993, 0.995, 0.997); 0.9]}
= {1(-) [(0.95270, 0.96250, 0.96934); 0.6], [(0.94596, 0.96250, 0.97818); 0.7]}
= [(0.03066, 0.03750, 0.04730); 0.6], [(0.02182, 0.03750, 0.05404); 0.7]

On the basis of above calculations, we find that the failure interval of "Power failure system" as following

= [(0.0466, 0.03750, 0.04030); 0.6], [(0.03182, 0.03750, 0.04404); 0.7]

Thus the reliability interval of "Power failure system" can be described as the following vague number.

[(0.9540, 0.96250, 0.96934); 0.6], [(0.9596, 0.96250, 0.97818); 0.7] (2.5.1)

Expression (2.5.1) interprets the reliability to lie in the interval (0.9540, 0.96934) with truth value 0.6 and in the interval (0.9596, 0.97818) with truth value 0.7. It can be observed that the crisp value of traditional reliability lies within the obtained intervals.

In order to find the vague importance index, we calculate P_T as the followings:

P_{T_C} = [(0.03061, 0.03741, 0.04705); 0.6], [(0.02181, 0.03741, 0.05376); 0.7]

$P_{T_{A_1}}$ = [(0.02872, 0.03267, 0.03962); 0.6], [(0.02182, 0.03267, 0.04449); 0.7]

$P_{T_{A_2}}$ = [(0.02579, 0.03169, 0.03962)0.6], [(0.01888, 0.03169, 0.04449); 0.7]

$P_{T_{B_1}}$ = [(0.02383, 0.03072, 0.03865); 0.6], [(0.01493, 0.03072, 0.04545); 0.7]

$P_{T_{B_2}}$ = [(0.02677, 0.03267, 0.04155); 0.6], [(0.01888, 0.03267, 0.04737); 0.7]

$P_{T_{D_1}}$ = [(0.02481, 0.03072, 0.03962); 0.6], [(0.01888, 0.03072, 0.04545); 0.7]

$P_{T_{D_2}}$ = [(0.02383, 0.02974, 0.03865); 0.6], [(0.01592, 0.02974, 0.04449); 0.7]

$P_{T_{E_1}}$ = [(0.03066, 0.03750, 0.04730); 0.6], [(0.02182, 0.03750, 0.05404); 0.8]

$P_{T_{E_2}}$ = [(0.03066, 0.03750, 0.04730); 0.6], [(0.02182, 0.03750, 0.05404); 0.8]

$P_{T_{F_1}}$ = [(0.03066, 0.03750, 0.04730); 0.6], [(0.02182, 0.03750, 0.05404); 0.8]

$P_{T_{F_2}}$ = [(0.03066, 0.03750, 0.04730); 0.6], [(0.02182, 0.03750, 0.05404); 0.7]

$P_{T_{G_1}}$ = [(0.03066, 0.03750, 0.04730); 0.6], [(0.02182, 0.03750, 0.05404); 0.7]

$P_{T_{G_2}}$ = [(0.03066, 0.03750, 0.04730); 0.6], [(0.02182, 0.03750, 0.05404); 0.7]

$P_{T_{G_3}}$ = [(0.02066, 0.02750, 0.03730); 0.6], [(0.01182, 0.02750, 0.04404); 0.7]

The I.F.I. of all basic events are calculated as following:

I (P_T, P_{TA1}) = 0.00068, I (P_T, P_{TA2}) = 0.024, I (P_T, P_{TC}) = 0.03085, I (P_T, P_{TZ}) = 0 I (P_T, P_{TB1}) = 0.03774, I (P_T, P_{TB2}) = 0.02408, I (P_T, P_{TD1}) = 0.03184, I (P_T, P_{TD2}) = 0.03869, I (P_T, P_{TE1}) = 0, I (P_T, P_{TE2}) = 0, I (P_T, P_{TF1}) = 0, I (P_T, P_{TF2}) = 0, I (P_T, P_{TG1}) = 0 I (P_T, P_{TG2}) = 0.00034, I (P_T, P_{TG3}) = 0.02184.

Basic events having vague importance index zero or a very small number indicate that those events play either no role or very negligible role in the top event. These events can therefore be ignored while calculating the crisp reliability using traditional method. In the present example, the fault tree given in Figure 3. Repeating the calculations of Section 3 after ignoring above events, one gets the crisp reliability estimate of the 'Model Failure System' to be 0.9899. Thus our approach of Fuzzy importance index could be useful in avoiding the underestimation of the reliability of the system.

5.Conclusion

A new Intuitionistic fault tree analysis model is proposed in this paper that modifies the fuzzy set arithmetic operations for implementing fault tree analysis. Proposed method leads to two interval estimates of reliability with different truth values. The reliability estimate obtained by traditional approach lies inside the intervals. This work also introduces the concept of Intuitionistic Fuzzy Importance Index that helps in discarding unimportant events from the classical fault tree analysis to avoid under/ over estimation of reliability. Results of Intuitionistic fault tree analysis are more flexible than the fuzzy fault tree analysis because the later method cannot describe the uncertainty of confidence level.

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