AN ANALYSIS OF EXISTING ARTIFICIAL STOCK MARKET MODELS FOR REPRESENTING BOMBAY STOCK EXCHANGE (BSE)

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Abstract - The goal of agent-based modeling of stock markets is to enrich our understanding of fundamental processes that appear in a market. Artificial stock markets are models of financial markets used to study and understand market dynamics. Agent Based Artificial Stock Markets can be seen as any market model in which prices are formed endogenously as a result of participants' interaction. There are various artificial stock markets in existence primarily for US and European markets. This study aims at assessing the suitability of existing agent based artificial stock market models for the Bombay Stock Exchange.

We analyze few continuous session artificial stock market model, namely Continuous Time Asynchronous Model (CTAM),Electronic Market Maker (EMM) Model, Continuous Extended Glosten Milgrom Model (CEGM) and KapSyn. We analyze their features and design to enable us assess their suitability to depict the BSE based on a few important parameters. It appears that the elements of KapSyn would be most suitable for representing the BSE, after incorporating modifications to cater for the absence of market makers in BSE.

Key words – agents; artificial stock markets; market makers; liquidity; fundamental value; price discovery; autonomy; asynchronous behavior; learning

I. INTRODUCTION

The agent-based approach considers a population of intelligent adaptive agents and lets them interact in order to maximize their financial performance[1,2,3,4,5,6]. Artificial stock markets play a vital role as they reflect the working of real stock markets and help us in understanding their dynamics. In Artificial Stock Markets (ASM), prices should emerge internally as a result of trading.

Bombay Stock Exchange (BSE) is a continuous sessions market. Hence in this paper, few continuous quote-driven Artificial Stock Market Models have been analyzed and a comparative description on them has been delivered. The ASM under this study are: Continuous Time Asynchronous Model (CTAM), Electronic Market Maker (EMM) Model, Continuous Extended Glosten Milgrom Model (CEGM) and KapSyn.

II. Continuous Time Asynchronous Model (CTAM)

A Introduction

Continuous Time Asynchronous Model (CTAM) is a continuous, asynchronous model where individual traders interact[7]. This model is based on the microscopic simulation approach. Applying event-based simulation, continuous, asynchronous interaction is achieved. The agents are characterized to 'sleep' and 'wake up' at predefined times or due to previous decisions made. No central mechanism controls agents and thus each investor may play a different action at the same point in time. The trading sessions are continuous and quote driven(hence the presence of a market maker). Multiple investors and one market maker, who ensures liquidity of stocks and execution of orders are present. One stock which doesn't pay dividends is traded and investors place orders for that single stock. The stock has an underlying fundamental value, computed exogenously (i.e.), the market is unaware of it.

The fundamental value at the time t is V_t. It is constant most of the time and changes occasionally (i.e.) it follows a jump process. The jump process is modeled as a random process,

$$V_{t+1} = V_t + \omega\sim(0, \sigma) \qquad (1)$$

where, ω~ (0, σ) represents a sample from a normal distribution with mean zero and variance σ2.

B The investor

The investors are classified as informed and uninformed investors depending on the information regarding the fundamental value. Informed traders are further classified into perfectly informed and noisily informed. The perfectly informed knows the correct fundamental value and the noisily informed know the fundamental value with some noise,

$$W_t = V_t + \tilde{\psi}(0, \sigma_w) \qquad (2)$$

Here, $\tilde{\psi}(0, \sigma_w)$ represents a sample from a normal distribution with mean zero and variance σ_w^2 . Finally, uninformed traders do not know what the underlying fundamental value is, and they trade randomly. Based on their observations, the traders buy when their observed fundamental value is higher than the ask quote of the market maker and sell when their value is lower than the market maker bid quote. Uninformed traders buy and sell with an equal probability (η).

C The Market Maker

The fundamental value is unknown to the market maker and he tries to capture it by responding to the behavior of the traders. The market maker keeps a probability density estimate over a whole range of possible values for the stock, which is updated after the arrival of an order. The market maker knows some factors such as the fraction of informed (α) and uninformed traders (1-α), the probability with which the uniformed traders trade (η), the initial fundamental value (V_0),an occurrence of a jump process. The market makers carry out tasks such as: Receive and execute orders; Update the probability density estimate based on the received orders; Calculate the expected value of the stock based on the updated probability values; Adjust the bid and ask quotes according to the changes in the expected value.

The market maker sets the bid and asks prices such that they reflect the fundamental value of the stock. The bid (B) and ask (A) quotes are calculated as follows:

$$B = \frac{1}{P_{\text{Sell}}} \sum_{V_i=V_{\min}}^{V_{\max}} V_i \Pr(\text{Sell} \mid V = V_i) \Pr(V = V_i),$$

$$A = \frac{1}{P_{\text{Buy}}} \sum_{V_i=V_{\min}}^{V_{\max}} V_i \Pr(\text{Buy} \mid V = V_i) \Pr(V = V_i), \qquad (3)$$

where P_{Sell} is the a-priori probability of a sell order, and P_{Buy} is the a-priori probability of a buy order

D Market Price Discovery

- *Perfectly Informed Traders:* As the perfectly informed traders know the exact fundamental value, when the market maker releases the bid quote, all these traders place orders resulting in a condition called overshoots. When the market maker encounters such a condition he comes to know that the fundamental value is a value well above his bid quote. The market makers thus are always in line with the fundamental value except during the jump periods. On the contrary, the fact that a sell order is submitted helps the market maker understand that the current bid-ask spread being probably above the actual fundamental value. After this, the market maker's bid-ask spread is set around the actual fundamental value so the probabilities within the area between the bid and ask price are increased while the others are decreased, creating a high peak around the actual fundamental value, until another trade arrives.

- *Noisily Informed Traders:* In this case, for every update of the market maker, the probabilities for a whole range of values are taken into account because of the additional noise in the traders' decisions. The case is similar to that of the perfectly informed traders except for the observation of the fundamental value with some noise. Because the market maker has to rely on noisy information, the expected value most likely does not exactly follow the fundamental value, but the fundamental value with additional noise that is received by the noisily informed trader.

- *A combination of both perfectly and noisily informed traders:* Here the market maker takes more time to decide upon the fundamental value because the market now consists of both the groups. Large jumps are encountered often which at times is difficult to track.

E Important Issues

This model is continuous and asynchronous and works in line with the real financial markets; the herd like behavior of the traders reacting to a situation mitigates the versatility of the system and adapting the learning algorithm of the market maker to account for both perfectly informed and noisily informed traders is a challenge.

III. ELECTRONIC MARKET MAKER (EMM) MODEL

The papers[8,9]] presents an adaptive learning model for market-making under the reinforcement learning framework. Reinforcement learning is a learning technique in which agents aim to maximize the long-term accumulated rewards. No knowledge of the market environment, such as the order arrival or price process, is assumed. Instead, the agent learns from real-time market experience and develops explicit market-making strategies, achieving multiple objectives, including maximizing of profits and minimization of the bid-ask spread. Reinforcement learning can be considered as a model-free approximation of dynamic programming. Bid and ask prices are dynamically determined to maximize some long term objectives, such as expected profits or expected utility of profits. The knowledge of the underlying processes is not assumed, but learned from experience.

A The Model

In the basic model, the market-maker quotes a single price. The optimum strategies can be determined analytically and it can be shown that the reinforcement algorithms successfully converge to these strategies. The basic model is then extended to allow the market-maker to quote bid and ask prices. While the market-maker controls only the direction of the price in the basic model, it has to consider both the direction of the price as well as the size of the bid-ask spread in the extended model.

B Important Issues

This model can be applied to market situations for which no explicit model is available, since reinforcement learning assumes no knowledge of the underlying market environment. The main limitation of these models is that, price process and order arrival process have to be assumed. The environment state is only partially observable. The simulation results show initial success in bringing learning techniques to building market making algorithms.

IV. CONTINUOUS EXTENDED GLOSTEN MILGROM MODEL (CEGM)

This model is an extension of the Glosten Milgrom information based model[10], proposed to show the influence of informational asymmetry on the bid-ask spread, based on the learning market maker from Das[11][12]. The market maker tries to discover the fundamental value of a stock by means of Bayesian learning. He determines the bid and ask quotes based on his expectation of the real value, the order flow, taking into account his prior knowledge regarding the ratio of informed and uninformed traders. A nonparametric density estimation technique is used to maintain a probability distribution over a range of expected true values. The market-maker uses these probability estimates to set bid and ask prices. Discrete time simulation is applied in this extended model, as well as a probabilistic representation of the order flow.

A The Framework

Trading sessions are continuous and the execution system is quote-driven. There is one stock traded. One market maker and multiple investors are represented. The stock does not pay dividends. It is assumed that the stock has an underlying fundamental value, which is generated exogenously to the market. The underlying fundamental value of the stock follows a jump process, being constant most of the time.

B Trading

Trading is organized in trading rounds as a sequence of bilateral trading opportunities. Each trading opportunity involves a single potential investor who is selected at random from an unchanging pool of potential traders. The selected trader can buy at the offer, sell at the bid, or choose not to trade. The market maker is responsible for the liquidity of the stock and the execution of orders at the current bid or ask price. He sets bid and ask prices as a function of the order flow and the market information he possesses.

C Probability Density Estimate (PDE)

All orders are assumed to be market orders of one unit. The market maker does not know the fundamental value, but, in order to ensure an efficient market, he tries to capture it by maintaining a probability density estimate (PDE) over a range of expected true values. The probability estimates are initialized according to the normal distribution. The initial bid and ask prices are calculated from this initial PDE and a priori expectations of the market maker.

D Trading Rounds

After initialization, the trading rounds start. A round consists of the following steps: The probability is evaluated for a jump in the fundamental value, and the jump is carried out if it is the case; An investor is selected randomly from the pool to place an order; A Buy/ Sell/ No order is sent by the selected trader to the market maker; The market maker processes the order and carries out the transaction if it is the case; The market maker updates his probability density estimate of possible fundamental values and finally, the market maker updates the bid and ask prices. In the EGM model a jump in the fundamental value occurs with some probability (0.001 in the experiments) at every trading period, ie at every discrete point in time. The jump process is modeled as a random process

E The Investors' Behavior

Investors are differentiated based on the information they receive regarding the fundamental value. There are two types of investors, viz Informed traders and Uninformed traders. The informed traders are further classified as perfectly informed or noisily informed. Perfectly informed traders observe the correct fundamental value, while noisily informed investors observe a distorted fundamental value. Finally, uninformed traders do not know what the underlying fundamental value is, and they trade randomly. Informed traders decide whether to trade or not, based on their observation of the fundamental value. An informed trader will buy if the fundamental value that he observes is higher than the market maker's ask price. He will sell if the fundamental value that he observes is below the bid price. He will place no order if the observed fundamental value is within the bid-ask spread. Uninformed traders place buy and sell orders with equal probability. They can also decide not to place orders with probability

F The Behavior of the Market Maker

After an investor has been selected, and has placed an order, it is the market maker's task to carry out the rest of the actions within a trading round. On his turn, the market maker needs to carry out the following tasks: receive and execute orders; update the probability density estimate (PDE) based on the received orders and adjust the bid and ask quotes according to the changes in the PDE. The market maker executes sell orders at the current bid price and buy orders at the current ask price. The private information regarding the fundamental value is revealed implicitly by the type of the orders submitted by the (informed) traders. Information regarding the fundamental value of the stock diffuses from the informed traders to the market maker in this way. A series of sell orders might indicate that the fundamental value is lower than the current bid price, and a series of buy orders might indicate that the fundamental value is higher than the current ask price. However, the market maker will have to take into account the noise incorporated by the orders of the noisily informed traders, and the noise implied by the orders submitted by the uninformed traders. The market maker aims to set bid and ask prices to capture the underlying fundamental value of the stock. The fundamental value is known only by the (perfectly) informed investors, and is not known by the market maker. The main task of the market maker is to learn this value. As mentioned above, he tries to do this by maintaining a set of possible true values with probability estimates attached to each of them. The range of possible values, the corresponding probabilities, and the learning process of the market maker is based on a set of current and a-priori known information.

G Important Issues

This model consist of “informed" trading agents who decide to trade based on received signals of the true or fundamental value of the stock, and “uninformed" trading agents who trade for exogenous reasons and are modeled as buying and selling stock randomly. The combination of traders and market-making agents replicate properties observed in real financial markets. The bid-ask spread increases in response to uncertainty about the true value of a stock. Average spreads tend to be higher in more volatile markets; and market-makers with lower average spreads perform better in competitive environments.

V. KAPSYN

The KapSyn ASM[13] presents an approach to calculating transaction costs that is based on the capital market’s microstructure. KapSyn can imitate the microstructure of any stock exchange, especially with respect to its price-finding procedures. Since trading has increasingly been concentrating on cutting costs, several alternatives have evolved for placing orders, and transaction costs have become increasingly important as a means of evaluating them. Investors are represented at individual level. The model applies a Markov process with continuous time and discrete state space to model the capital market’s dynamics. It uses a stochastic utility approach to evaluate the traders utility. Like real stock exchanges, traders are able to place orders or revise their orders. The modeling of intra-day market simulates trading processes of a realistic environment. At any point during the trading session the market participants can submit bids and asks. Whenever two orders match in volume and price, the quote is fixed. Under certain conditions, an auction may take place. Every trader believes that, at the end of the planning period, the price of a security will equal the individual estimate. The basic rule of trading: traders aim to buy securities whose share price is above their individual intrinsic value estimates, and they aim to sell those securities whose share price is below their individual estimates and invest the proceeds elsewhere. Transaction costs and the opportunity costs of alternative investments also have to be taken into account.

Brokers are also represented. More than one stock is traded on the KapSyn. Investors have actions at their disposal associated to each stock. An action can be placing a buy order, placing a sell order or changing the expected value. With each action a reaction time is associated. The reaction time depends on the expected utility that the action can generate and is lower for higher utilities. At each simulation round an action is selected for execution. Selection depends on the reaction time associated to the actions. Actions with smaller reaction times have a higher probability to be selected. The continuous time is implemented by using a discrete state space. Transition from one state into another depends on time related parameters. The agents’ decision process regarding which action to take next is independent of each other and takes place simultaneously in parallel. A decision process contains the following steps: each agent determines a set of alternative actions for each stock; each agent selects one action for each stock and finally, one action is selected for each agent.

All the selections depend on the utility and execution time of the actions. The greater the expected benefit gained from the action of an agent, the shorter will be the reaction time and the higher the selection probability. After all agents have made their decision with respect to the next action to be carried out, the system executes the action with the shortest reaction time. After an action that is observable by all market participants, i.e. a buy or a sell action, the decision process determining the next action is repeated on the basis of the new market conditions. Therefore, in this case, only one action is executed in a simulation round.

A Important Issues

In KapSyn, continuous time is implemented by using a discrete state space. KapSyn is the only ASM in which intra-day data is generated, brokers are represented and more than one stock is traded.

VI. A COMPARATIVE ASSESSMENT

CTAM,EGM, EMM and KapSyn are artificial markets that represent continuous quote-driven trading orders wherein market makers set bid and ask quotes. In BSE, however, there is no market maker at present. The design and mechanism of these ASMs is analyzed based on the aspects of structure of the model and mechanism of formation of price[14].

A Assets

ASMs trade only two types of assets viz one risk-free and one risky stock. Risk free assets might represent PPF or Government bonds paying a constant interest rate. In Kapsyn multiple (risky) assets can be traded. A well-determined fundamental value exists in most of the ASMs. This value typically follows a random IID (Independent and Identically Distributed) process. In KapSyn there is no unique fundamental value.

B Orders

Trading by limit orders or both limit and market orders can be placed. Both limit and market orders are placed in CTAM, EGM and EMM. In EGM and EMM, investors' give market orders, and the market maker determines bid and ask quotes with limit prices. In the KapSyn market, limit orders are generated, but traders might also decide to accept quoted orders. Unexecuted orders at CTAM and KapSyn can be canceled.

C Market Participants

Individual investors are simulated in all ASMs. Traders place orders as the result of some decision that is mainly utility maximization in every trading round. In this way only investors are represented by these traders. The behavior of market makers is modeled in EGM, EMM and Kapsyn. In CTAM and KapSyn, individual investors with their own decision-making behavior are simulated. Brokers are modeled only in KapSyn.

D Trading Sessions

At the KapSyn stock market continuous time is implemented by using a discrete state space. At CTAM, discrete event simulation is applied in order to model continuous trading sessions. Price formation is based on automated central execution systems. In KapSyn model, price formation depends on the microstructure of the stock exchange modeled.

E Investor Objectives

The objectives of the investors are to maximize profits and depend on the strategy they use: fundamental or technical: EGM and EMM is through arbitrage opportunities; KapSyn and CTAM is by maximizing a utility function and wealth respectively.

F Risk

Investors' risk profile is modeled by some ASMs by introducing risk averse traders. CTAM strive to minimize risk, whereas in KapSyn the risk is measured as the deviation of the actual portfolio from the desired portfolio.

G Investment Methodology

Investment methodology differ and investors are classified as Fundamentalist traders(CTAM, EMM and EGM); Technical traders(CTAM and Kapsyn and Random traders(CTAM, EGM and EMM).

H Trade

The required weight of the stocks usually depends on the utility function applied. In CTAM shares to trade is based on forecast and a fraction of the wealth. The traders decide to sell/buy if the expected price is below/above the current market price. And stochastic functions are applied to determine trading. Buy and sell orders are placed with equal probability in EGM and for random traders in CTAM. At KapSyn the probability of being selected is related to the expected utility. In CTAM, for the limit orders, the limit price defined by the traders is the forecasted price of the stock.

I Adaptation

Adaptations vary from simple value adjustments to intricate evolution of strategies. An agent monitors the new market prices; perceived fundamental values; bid and ask quotes when applicable; and the performances. In KapSyn, technical traders correct the expected value of a stock upward or downward.

J Time Horizon

The time-horizon of the investment objectives hold during the whole experiment. All traders simultaneously make a trading decision whereby it usually results in placing a new order. Asynchronous behavior is modeled by selecting only a fraction of traders at any time. In every simulation round usually one order is generated thereby depicting the asynchronous behavior of traders. At KapSyn the next event to be carried out depends on the reaction time of all events prepared by investors. In CTAM the timing for placing an order can be predefined by some event. Timing of orders in CTAM and KapSyn is determined by the investors and not by the system centrally, thereby representing autonomy of traders.

K Equilibrium Price

In KapSyn, equilibrium is determined at the price at which trading volume is maximized. The behavior of the market makers is adaptive: in EGM it is Bayesian learning and EMM it is reinforcement learning. The market makers at KapSyn modify their quotes with respect to the limit order book.In EMM the electronic market maker reacts when the order imbalance reaches a predefined threshold.

L Autonomy

Most of the traders do not decide themselves whether they will trade in the next trading cycle or not. Instead, they are selected globally by the system. In CTAM traders react to some events or wait for a while before they analyze the market conditions. In KapSyn a reaction time is associated to each individually planned action as a function of their utility. All investors are goal-directed, except for cases in which decisions are represented by random order flows. Interactive behavior occurs since the agents need to trade with each other. Learning is Bayesian learning in EGM, and AI based techniques in EMM.

VII. FINDINGS

We see that there are many ways to model the behavior of traders. Table 1 gives a comparative study of important parameters having a predominant impact on the market dynamics. In KapSyn and CTAM, continuous order matching and asynchronous decision-making is done, which suits the mechanism prevailing at BSE. Representations of traders in most ASMs illustrate passive agents, while traders on the BSE are autonomous. Autonomous behavior can only be accomplished if the agents themselves decide when they want to trade. Most ASMs focus on representing only the investor, thereby leaving out the brokers. In reality, however, the behavior of brokers directly affects price formation. Only KapSyn includes brokers. Representations of traders in ASMs illustrate passive agents, while agents on the BSE market are autonomous. The most detailed, representation of stock exchanges is implemented by the KapSyn model. Price formation in this model depends on the microstructure of the stock exchange that is modeled. Hence, the KapSyn ASM model would be most suitable to represent the BSE. However, in this model, a market maker plays a prominent role, where as, in the BSE, the role of the market maker does not exist.

VIII. CONCLUSION

This study aims at analyzing existing agent based, continuous sessions, artificial stock market models, in order to assess its suitability to study the Bombay Stock Exchange (BSE). In the BSE, trading occurs in continuous asynchronous sessions and traders are autonomous. We have compared the characteristics of few artificial stock market models, namely Continuous Time Asynchronous Model (CTAM), Electronic Market Maker (EMM) Model, Continuous Extended Glosten Milgrom Model (CEGM) and KapSyn. There are many ways to represent the aspects of price formation and trading behavior. Most ASMs focus on representing only the investor. In reality, however, the behavior of brokers directly affects price formation. Only KapSyn includes brokers. It appears that the elements of KapSyn would be most suitable for representing the BSE. However, in this model, a market maker plays a prominent role, where as in the BSE, the role of the market maker does not exist. In case KapSyn ASM is planned to be adopted for the BSE, corresponding modifications to take care of this aspect would have to be incorporated in the model.

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TABLE I: A COMPARISON OF IMPORTANT PARAMETERS OF ASMS

PARAMETERS	CTAM	EMM	CEGM	KAPSYN
TRADED ASSETS	RISKY AND RISK FREE.	RISKY AND RISK FREE	ONLY RISKY.	RISK FREE AND RISKY(N<123)
TRADING SESSION	CONTINUOUS (SLEEP & WAKE)	CONTINUOUS	TURN BASED AND AUTONOMOUS	CALL AND CONTINUOUS
INVESTMENT OBJECTIVE	OWN MAXIMUM NO. OF INSTRUMENTS	LIQUIDITY	LIQUIDITY	MAXIMISE UTILITY
INVESTMENT STRATEGY	BASED ON THE KNOWLEDGE OF FUNDAMENTAL VALUE	FIXED SET OF STRATEGIES	BASED ON THE KNOWLEDGE OF FUNDAMENTAL VALUE	INDIVIDUAL EXPECTATION
MARKET PARTICIPANTS	INVESTORS MM	INVESTORS MM	INVESTORS MM	INVESTORS –N<123 BROKERS MM