

# Optimum Design of Composite Plates under Thermal Buckling Loads using Imperialist Competitive Algorithm

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**Abstract**—Thermal buckling loads of laminated composite plates are maximized for a given total thickness. Fiber directions and relative thickness of layers are considered as design variables. Analysis of buckling temperature is carried out by using the finite element method, while the imperialist competitive algorithm (ICA) is employed to optimize as many as seven variables for the different layered plates. The imperialist competitive algorithm (ICA) is thus evaluated for its recognition.

**Keywords**—composite plates; imperialist competitive algorithm (ICA); finite element analysis.

## I. INTRODUCTION

Fiber-reinforced composite materials are widely used in mechanical and material engineering applications for reducing structural weight due to the high specific stiffness and strength of the composites. However, a change from design for minimal weight to design for cost has become crucial during recent years [1–3]. From the standpoint of strength alone, many researchers [4–6] have studied the thermal buckling behavior of laminate plates.

Global optimization problems are finding many applications in science and engineering. So far, evolutionary algorithms [7, 8] have been proposed for solving global optimization problems. Walker investigated [9] the optimal design of laminated composite plates for maximum thermal buckling load with simply supported edge conditions. Later, Maloy studied [10] on optimum design of laminated composite plates using by genetic algorithm (GA) for maximum thermal buckling loads.

Recently, a novel evolutionary algorithm was proposed [11] in 2007. The Imperialist Competitive Algorithm was inspired from a socio-political phenomenon, and has been used for optimization of transmission conditions of thin adhesive layer in [12]. Mozafari has optimized [13] composite plates by using the ICA.

In this paper, the optimization of composite plates for thermal buckling loads was investigated based on imperialist competitive algorithm (ICA).

## II. FINITE ELEMENT FORMULATION

The third order shear deformation theory is used for analyzing the following displacement fields,

$$\begin{aligned} u(x, y, z) &= u_0(x, y) + z u_1(x, y) + z^2 u_2(x, y) + z^3 u_3(x, y), \\ v(x, y, z) &= v_0(x, y) + z v_1(x, y) + z^2 v_2(x, y) + z^3 v_3(x, y), \\ w(x, y, z) &= w_0(x, y). \end{aligned} \quad (1)$$

In Cartesian coordinates  $(x, y, z)$  the general strain–displacement relations are,

$$\varepsilon_x = u_{,x} + \frac{1}{2} w_{,x}^2, \quad \varepsilon_y = v_{,y} + \frac{1}{2} w_{,y}^2,$$

$$\begin{aligned}\varepsilon_{xy} &= u_{,y} + v_{,x} + w_{,x}w_{,y}, \quad \varepsilon_{xz} = u_{,z} + w_{,x}, \\ \varepsilon_{yz} &= v_{,z} + w_{,y}.\end{aligned}\quad (2)$$

where  $\varepsilon_x$  and  $\varepsilon_y$  are the normal strains, and  $\varepsilon_{xy}$ ,  $\varepsilon_{xz}$ , and  $\varepsilon_{yz}$  are the shear strains. Here  $u, v$  and  $w$  (change to  $u, v, w$ ) denote the displacement components in the  $x, y$ , and  $z$  directions, respectively. Also,  $(,)$  indicates the partial derivative.

The Equation between the relative stress resultant, moment resultant and strain can be written as,

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} [K_{11}] & [K_{12}] \\ [K_{21}] & [K_{22}] \end{bmatrix} \begin{Bmatrix} \varepsilon' \\ R' \end{Bmatrix} - \begin{Bmatrix} N^T \\ M^T \end{Bmatrix} \quad (3)$$

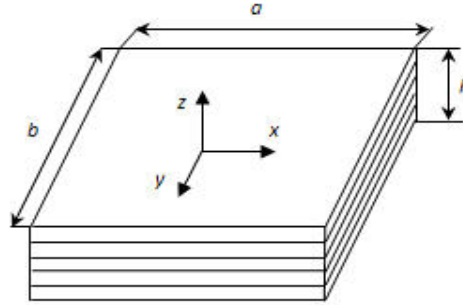


Figure 1: Illustration of specimen plate

where  $N$  and  $M$  are the relative stress resultants and moment resultants on the plate,  $\varepsilon'$  is strain,  $R'$  is the middle surface curvatures and  $[K_{ij}]$  is extensional stiffness matrices of composite layers, respectively.

The vector of thermal load can be introduced as:

$$\{N^T, M^T\} = \sum_{\lambda=1}^n [Q_{ij}]_{\lambda} \{\alpha\}_{\lambda} z t(z) (1/z) dz \quad (4)$$

where  $[Q_{ij}]_{\lambda}$  is transformed plate stress to reduced stiffness matrix of the  $\lambda^{th}$  lamina.

In this section the stepping FEM is employed to validate the optimization result. The commercial finite element code MSC. Marc is used for the simulation of the thermal buckling behavior of laminate composite plate. The 2D finite element mesh is built up of four-node elements (see Fig. 1).

### III. IMPERIALIST COMPETITIVE ALGORITHM

The Imperialist competitive algorithm (ICA) is a new progressive algorithm for optimization, and the flowchart of ICA is shown in Fig. 2.

This algorithm starts with an initial population. Each population in ICA is called country. Some of the best countries in the population are selected to be the imperialists and the rest form the colonies of these imperialists. In this algorithm the more powerful the imperialists, the more they have colonies. When the competition starts, the imperialists attempt to achieve more colonies and the colonies start to move toward their imperialists. So during the competition the powerful imperialists will survive and the weak ones will be collapsed. At the end just one imperialist will remain. Further details about this algorithm are described in [11].

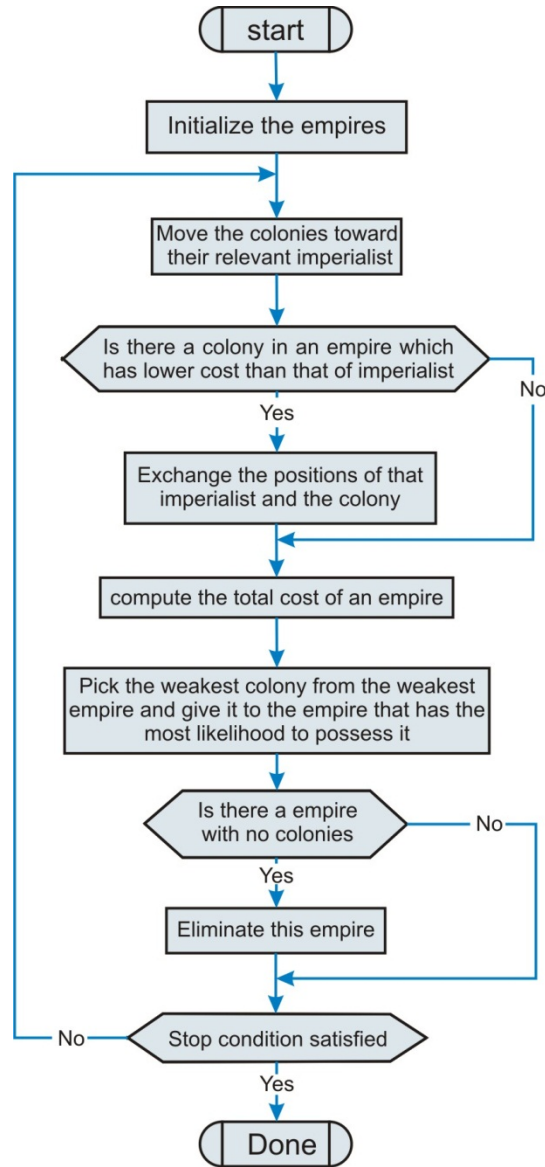


Figure 2. Illustration of imperialist of competitive algorithm (ICA)

The present problem is to maximize the buckling temperature  $Opt. T_{cr}$  for the optimum combination of ply-angles  $\theta_1, \theta_2, \theta_3 \dots \theta_n$  and optimum ply-thicknesses  $t_1, t_2, t_3 \dots t_n$  of the laminated plate having  $n$  layers.

To obtain an optimal design which considers both angle of ply and thickness of laminate plate, the objective function is defined as follows:

Maximize  $T_{cr}$

Subject to constraints,

$$0.04 \leq t_i / h \leq 0.9 \quad (5)$$

$$-90^\circ \leq \theta_i \leq +90^\circ \quad i = 1, 2, 3, \dots, n$$

The total thickness of the plate  $h$  is constant which can be written as  $\sum_{i=1}^n t_i / h = 1$ . In this optimization problem the goal function is the inverse of Equation (5).

## IV. RESULTS AND DISCUSSION

The Imperialist competitive algorithm (ICA) is used to optimize the design variables  $\theta_i$  and  $t_i$  which are the ply angles and thicknesses, respectively. The material properties of the present study are  $E_1 = 141 \text{ GP}$ ,  $E_2 = 31 \text{ GP}$ ,  $G_{12} = 9.31 \text{ GP}$ ,  $\mu_{12} = 0.28$ ,  $\alpha_1 = 0.018 \times 10^{-6} / \text{deg C}$  and  $\alpha_2 = 21.6 \times 10^{-6} / \text{deg C}$ .

Tables 1 and 2 show the comparison of the optimum ply angle using ICA with other references under simply supported and clamped edge conditions.

TABLE I. COMPARISON OF OPTIMUM PLY-ANGLE, WITH SIMPLY SUPPORTED PLATE

Angle	Ref [9]	Opt. GA [Ref 10]	Opt. ICA
$\theta$	45.1°	45.0°	49.8°

TABLE II. COMPARISON OF OPTIMUM PLY-ANGLE, WITH CLAMPED PLATE

Angle	Ref [9]	Opt. GA [Ref 10]	Opt. ICA
$\theta$	54.3°	52.9°	52.1°

Thermal buckling loads optimization results ICA are calculated with simply supported and clamped conditions which are illustrated Tables 3 and 4 and the results are compared with the other optimization method of genetic algorithm (GA).

TABLE III. COMPARISON OF OPTIMIZED BUCKLING LOADS, WITH SIMPLY SUPPORTED PLATE

$a / b$	Opt. GA Ref [10]	Opt. ICA
1	50.68°C	52.31°C
1.5	32.19°C	33.89°C
2	27.35°C	27.93°C

TABLE IV. COMPARISON OF OPTIMIZED BUCKLING LOADS, WITH SIMPLY SUPPORTED PLATE

$a / b$	Opt. GA Ref [10]	Opt. ICA
1	113.49°C	115.23°C
1.5	101.35°C	1.2.47°C
2	96.61°C	97.1°C

Table 5 shows the optimum design of thermal buckling loads, ply-angles and thicknesses of laminated composite plate for seven layered simply supported plates. The results of maximum buckling temperatures are compared for the different layers with simply supported plates [see Table 6].

TABLE V. OPTIMUM DESIGN OF MAXIMIZATION OF THERMAL BUCKLING LOAD

$a / b$	Angle ( $\theta$ )	Thickness ( $t$ )	Thermal buckling Load ( $T_{cr}$ )
1	[45 0 60 -45] <sub>s</sub>	0.33	71.3°C
1.5	[60 0 30 -60] <sub>s</sub>	0.21	59.8°C
2	[-30 45 0 30] <sub>s</sub>	0.18	51.4°C

Optimum solutions are obtained under simple tensile loading in  $x$ -direction which is shown in Table III.

TABLE VI. COMPARISON OF MAXIMUM BUCKLING TEMPERATURES (°C) FOR DIFFERENT LAYERS

Number of layers	$a / b = 1$		$a / b = 1.5$		$a / b = 2$	
	Ref [10]	Opt. ICA	Ref [10]	Opt. ICA	Ref [10]	Opt. ICA
3	69.03	69.48	61.21	61.32	61.20	61.29
4	69.3	69.71	60.43	61.39	60.16	61.37
5	69.4	69.85	60.87	61.48	63.76	61.45

## V. CONCLUSION

In this paper, an imperialist competitive algorithm (ICA) procedure has been developed for optimizing the laminate composite plates subjected to thermal buckling loads. The optimizations of the design variables showed good performance of ICA in laminate composite plates which are optimized for ply-angle and thicknesses of plates for maximum buckling temperatures.

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